THE ELEMENTS

OF

PRACTICAL HYDRAULICS;

FOR THE USE OF

STUDENTS IN ENGINEERING.

BY

SAMUEL DOWNING, M. A.,

PROFESSOR OF CIVIL ENGINEERING IN THE UNIVERSITY OF DUBLIN

LONDON:

LONGMAN, BROWN, GREEN, AND LONGMANS.

1855.

DUBLIN:
Printed at the University Press,
BY M. H. GILL.

INTRODUCTION.

The following work is intended as a Text-book for the Students in the School of Engineering, Trinity College, Dublin, in that branch of their intended profession which has reference to Practical Hydraulics.

The first two chapters are in a great degree a translation of D'Aubuisson's well-known work, "Traitè d'Hydraulique a l'usage des Ingenieurs," which has obtained extensive circulation on the Continent, as well as in England and America. The third chapter, relating to the flow of water in artificial channels, rivers, and pipes, is founded on the formula for the uniform motion of water, which is in very general use amongst English engineers, and possesses great advantages over that given by D'Aubuisson, in simplicity and facility of application. Indeed, we do not find that this more complex formula has obtained the full confidence of foreign engineers themselves. Thus M. Girard, who constructed the navigable channel intended to supply the street fountains of Paris with water derived from the river l'Ourcq—being in possession of all the necessary dimensions and data, except the rate of inclination of the surface in the longitudinal section—calculated this last by that more complicated formula of which we speak, but finally decided upon constructing the line with a fall nearly double that thus obtained (§ 120). The formula given in the third chapter has, moreover, served to determine the proportions and dimensions of many English works of the greatest

magnitude, which have, when completed, been found to fulfil perfectly their intended objects, and consequently it may be considered as based on an experience the most reliable, and, of all others, the most valuable to the civil engineer.

Throughout this work, the only dimension used is the foot and the cubic foot. We have in English works on Hydraulics a great variety of units: the gallon, the cubic foot, the ton, the cubic yard; and for length, the yard, foot, and inch. As soon as the student has become familiar with the value of the inches in a foot expressed decimally, it is hoped that this arrangement will be found useful. Of the eleven decimal fractions for the inches in a foot, five are well known, and the rest may be readily remembered:—

Inches.	Foot.	Inches.	Foot.
1	0.0833	7	0.5833
2	0.1666	8	0.6666
3	0.2500	9	0.7500
4	0.3333	10	0.8333
5	0.4166	11	0.9166
6	0.5000	12	1.0000

The Author takes this opportunity of returning thanks to the Board, for the liberality they have shown in defraying the greater part of the expenses of this Work.

CONTENTS.

ARTS		PAGE
1-5.	DEFINITIONS, &c.,	1-2

CHAPTER I.

DISCHARGE OF WATER FROM A RESERVOIR, THE SURFACE BEING AT A CONSTANT LEVEL.

6-7.	Divisions of the subject,	3-4
8-12.	Torricelli's Theorem, &c.,	4-8
13.	The Discharge—Theoretic Discharge,	8
14-18.	Actual Discharge, Coefficients, &c.,	8-12
19-33.	Discharge through a thin Plate, &c.,	12-22
34-36.	" " Cylindrical Adjutages,	23-24
37-41.	" " Conical Converging Adjutages,	25-29
42-43.	" " " Diverging Adjutages,	29-30
44-52.	Discharge under very small Heads,	30-35
53-82.	Flow of Water over Waste Boards, Weirs, &c.,	35-61

CHAPTER II.

DISCHARGE OF WATER UNDER A VARIABLE HEAD.

83-85.	Discharge of Water when the Level is variable upon one or both faces of the Orifice of Discharge,	62-63
86-90.	Descent of the Surface, a uniformly retarded Motion, &c.,	63-66
91.	Mean Hydraulic Charge,	66
92.	Reservoir receiving a constant supply during the descent of the Surface,	67
93.	Discharge taking place over a Weir,	68
94.	Reservoir not being Prismatic,	68-69
95-96.	Discharge from one Reservoir into another, &c.,	69-71
97-109.	Examples and Practical Applications to Chapters I. and II.,	72-104

CHAPTER III.

FLOW OF WATER THROUGH PIPES, ARTIFICIAL CHANNELS, AND RIVERS.

Arts.		Page.
110.	Accelerating Force,	105
111.	Laws of Friction of Solid Bodies,	106
112-114.	" " of Fluids, and Resulting Formulæ for Pipes and Channels, &c.,	106–113
115.	Best form of Transverse Section of Channels,	114
116–118.	Mean Velocity, &c.,	115-116
119–120.	Examples and Practical Applications,	117-124

INTRODUCTION.

1. The science of Hydraulics has for its object the knowledge of the phenomena of fluids in motion, and of the laws which regulate the production of these phenomena.

Applied as an art, its object is to render this knowledge available in the designs of the civil engineer, as in the determination of the dimensions of pipes for conveying water, gas, or air, and also in works for the collecting, conveying, and distributing the necessary supply of water, for mill-power, or for the summit levels of canals; or for the supply of cities; and, more generally, of all such works as depend for their suitable construction and proportions upon the result of calculations requiring a knowledge of the pressure and motion of fluids.

2. Fluids are defined to be bodies whose particles, by reason of their extreme mobility, yield to every the least force; they have, however, a certain degree of adherence or viscidity which binds them together. These bodies are divided into two classes, the incompressible, or fluids properly so called, and the elastic. Water is the more common type of the former, and the atmosphere of the latter.

3. Although all fluids, as indeed every substance in nature, is in strictness both compressible and elastic, yet the difference in degree is so marked, and the distinction in the expression of their laws so essential under this point of view, that this division must be retained, and hence we have Hydraulics, properly so called, treating of the incompressible fluids, especially water, and Aerometry, treating of the atmosphere and gases.

4. Before entering on the former we must state the values of two quantities which occur in all calculations in Hydraulics, namely, the weight of water, and the measure of the force of gravity.

In statements of this nature we should carefully distinguish between those that are mere arbitrary definitions and those which are but inferences from the definition, and also those which are natural quantities, determined by experiments. A cubic foot of water weighs 62.32106 lbs. avoirdupois, the foot being the third part of a brass bar, constructed by Mr. Bird about the year 1760 from the mean of several old and inconsistent standards then existing, and which bar was in 1824 declared by Act of Parliament to be the unit of measures of length, and named the "Imperial Standard Yard." The pound avoirdupois was by the same Act defined to be 7000 grs., the grain being thus determined:—A weight of two pounds Troy having been constructed by the same artist that made the standard yard above mentioned, the half of it (one pound Troy) was divided into 5760 grs. (12 oz. × 20 dwts. × 24 grs.), and 7000 of these grains are equal to one pound avoirdupois. Measures of capacity have for their unit the gallon, which was defined to contain 10 lbs. av. weight of water,—the thermometer at 62° Fahr., and the barometer standing at 30 inches. A cubic inch of water, at the same temperature and pressure, was found by experiment to weigh in air 252.458 grs.; and hence, if we divide 70000 by 252.458, we shall have the number of cubic inches in the imperial gallon, which, therefore, is equal to 277.274 cubic inches; and if we multiply 252.458 by 1728, the number of cubic inches in one cubic foot, we have, as above, 62.32106 lbs.: hence, also, 6.23 imperial gallons to one cubic foot.

The cubic foot of water is, in practice, supposed to be equal to 62.5 lbs. av., or 1000 ozs. at 16 ozs. to the lb.; and we also have 36 cubic feet to a ton, or 6 tons to a cubic fathom.

The measure of the force of gravity is the velocity acquired in one second by a body falling freely from a state of rest, and is equal to 32.1948 feet per second, and always denoted by the letter g.

5. The Mètre adopted in France in 1798 as the unit of lineal measures is equal to 39.37079 inches. It is therefore 3.28089 feet, and is subdivided decimally into the decimetre, the centimetre, and the millimètre; and in order to assist the forming a clear idea of the comparative value of the French and English lineal measures, they have been drawn on Plate I. from a common zero. The cubic decimetre of water at 39.38° F., and barometer at 29.922 inches, is the standard of weight named the Kilogramme, = 2.20485 lbs. av. — *Vide* Galbraith and Haughton's Arithmetic, 2nd edition, Tables of Foreign Weights and Measures.

ELEMENTS OF HYDRAULICS.

CHAPTER I.

ON THE FLOW, THROUGH AN ORIFICE, OF WATER CONTAINED IN A VESSEL.

6. THE vessel from whence the water issues may be, first, maintained at a constant height of surface; or secondly, it may receive no supply, and consequently be exhausted; or thirdly, the orifice, instead of discharging into the air, may do so into another reservoir, under more or less resisting counter-pressure; and hence *three divisions* of this part of the subject.

7. The opening through which the water flows is placed either in the bottom or in one of the sides of the experimental tank, most generally the latter, in which case the surface of the water in the basin should be above the upper edge of the orifice: this orifice is opened either in a thin plate,—that is to say, in a plate whose thickness is at the most less than the half of the diameter or smallest dimension of the orifice,—or it is furnished with an adjutage, or short funnel or pipe, sometimes cylindrical, sometimes conical, converging towards an external point, less often diverging. An orifice placed in a very thick plate would evidently be equivalent to one of the same diameter if placed in a thin plate with an adjutage attached. We may also have the surface of the fluid below the upper edge of the orifice; that part of the border or circumference is then as if it did not exist, and very frequently it is not applied, and the opening is then unlimited on its upper part, and is called an overfall (deversoir). The laws of the flow of water in this second case offer some peculiarities, and form the subject of a separate investigation. When the surface reaches but up to a very small height above the opening, we also have special circumstances: this case is intermediate between the two others first mentioned.

Before entering upon them, it is necessary to state briefly the general principles of the flow of water, and the modifi-

cations which the "contraction" of the fluid vein suffers in passing through the various orifices to be noticed. The vertical distance of the surface of the fluid above the centre of gravity of the orifice is called the *charge* of the water upon the orifice, or the head under which the flow takes place.

8. Let a vessel X (Fig. 1), maintained constantly full of water up to the level AB, have upon the horizontal faces CD and EF the orifices M and N, the fluid will issue in vertical jets, which will rise almost to the level of the water AK: they would rise fully up to it but for the operation of causes about to be detailed. Now, by the first principles of Dynamics, in order that a body impelled in a vertical direction should reach to any height, it is necessary that at the point of departure it should have had a velocity equal to that which it would have acquired in falling freely from that height; consequently the particles of the fluid must have had a velocity nearly equal to that due to the charge—that is, to the height of the surface of the water above the orifices.

So also, if upon a vertical face BR an orifice O be placed, we shall see hereafter (§ 31) that from the respective values of the lines OP and PQ the fluid must have issued from O with a velocity due to the height OB. It would issue with a velocity due to BR if the orifice had been opened in the bottom RT of the vessel; and the velocity is the same in O, in O_1, and O_2, the directions being different, but the charge the same. This truth holds good for different orifices, whatever may be the ratio of the area of the orifice to the horizontal section of the water in the vessel, provided that the level of the water is kept constantly the same, and tranquil; which last, however, cannot be attained if the orifice be large in proportion to that surface—the water of the supply, in that case, producing disturbing movements in the reservoir.

Thus the velocity acquired by a body falling freely by the force of gravitation from the height H is equal to that of the fluid as it issues from the orifice; that is—

$$V = \sqrt{2gH},$$

in which H is the "charge," and g the dynamical measure of the force of gravitation, being the rate at which a falling body is moving at the end of the first second.

9. Now, although this perfect equality does not strictly exist, yet experiments show that V is proportional to $\sqrt{H}$. The following Table of the results of experiments by Castel, D'Aubuisson, Bossut, Poncelet, &c., exhibit this truth:—

Diameter of the Orifice.	Charge above the Orifice.	Series of	
		Square Roots of the Charges.	Discharges or Velocities.
Feet.	Feet.		
0.0328	0.085	1.000	1.000
	0.098	1.074	1.064
	0.131	1.241	1.244
	0.164	1.386	1.393
	0.196	1.519	1.524
0.088	4.265	1.000	1.000
	9.580	1.500	1.497
	12.500	1.713	1.707
0.265	7.677	1.000	1.000
	12.500	1.305	1.301
	22.179	1.738	1.692
0.531	6.922	1.000	1.000
	12.008	1.316	1.315
Square Orifice 0.656 by 0.656	1.312	1.000	1.000
	2.296	1.323	1.330
	3.281	1.581	1.590
	4.265	1.803	1.806
	5.249	2.000	2.000

It will be observed that the charges vary in the ratio of 1 to 200 and more, and the sections of the orifices from 1 to 500, and yet in all cases the velocities have followed the ratio of the square roots of the charges, the minute discrepancies, sometimes giving too great a number, and sometimes too small, may be neglected, being inseparable from experiments of this nature. The actual object of measurement in the experiments is the *quantity* discharged in a given time; but it is evident that, with the *same orifice*, the discharge is exactly proportional to the velocity with which the fluid issues, and, therefore, that column in the Table which expresses the gauged discharges, reference being made to some one discharge as a unit, also expresses the velocities.

10. The general principle, that the velocities are as the square roots of the charges, as also the theorem of Torricelli (§ 8) in the case in which it is applicable, extends to every kind of fluids,—to mercury, oils, alcohols, and even aeriform fluids; so that the velocity with which each of them issues from an orifice is independent of its particular nature: of its density, it depends solely on the charge. Experiment demonstrates this, and very simple reasoning suffices to show its truth. Take the case of

mercury: the particles situated immediately in front of the orifice, and in which it is necessary to create a certain velocity, are, it is true, fourteen times more dense than those of water, and they, consequently, oppose to motion a resistance fourteen times greater than it would do; but the mass also which presses upon these particles, and produces the velocity of exit, being greater in the same proportion, gives a motive force fourteen times greater. Thus a compensation exists, and the velocity impressed remains the same; and in like manner it may be proved for a fluid lighter than water.

11. The proposition that has now been laid down with respect to the velocity of water issuing through an orifice is equally true in cases when the discharge takes place *in vacuo*, as when in the atmosphere, the velocity is always the same, with the same head, whatever be the pressure upon the free surface of the water in the vessel, provided the jet of water at its exit from the orifice be subject to an equal exterior pressure. But the velocity will be very different from that due to H if the pressures be not equal upon these two surfaces.

If the pressure be greater against the orifice at A than upon the free surface of the water BC (Fig. 2), then the excess of the former above that on the free surface must be less than that of a column of the fluid whose height is the vertical distance of the orifice A below the surface BC, otherwise there could be no discharge. Let us, then, take an horizontal plane DE below the plane BC, and at such distance from it that the weight of the column of the water contained between the two planes, and whose base is the unit of surface, may be equal to the excess of pressure at A, of which we are speaking,—the water in the vessel having but a very slight degree of motion on account of the relatively small area of the orifice, which is always understood to subsist; and therefore we may assume the pressures to be transmitted as if the contained liquid was in equilibrium. The pressure, then, which exists upon any point in the plane DE will be equal to that upon any point in BC, *plus* the supposed excess of pressure against the exterior of the orifice; and, therefore, the pressure will be the same upon the plane DE and that against the orifice at A. The liquid below the plane DE is then in the same condition as if that contained between BC and DE were removed, and the free surface and exterior of the orifice were under equal pressures; and thus the same formula will represent the velocity:

$$V = \sqrt{2g\,(H - h_1)},$$

h_1 denoting the depth of the plane DE below BC.

12. If the exterior pressure on A were less than that upon the surface BC, we may conceive the excess of pressure upon it to be produced by a liquid of the same specific gravity as that in the vessel, applied above BC and terminating in a free surface D′E′, situated at such height that the vertical distance represents as before the column of the liquid whose pressure is equal to the excess of the pressure on BC above that against the orifice at A. The flow, then, will take place with the same velocity as if the free surface of the liquid, instead of being in the plane BC, and supporting this excess of pressure, were at D′E′ and supported the same pressure as the orifice at A; the formula will then be—

$$V = \sqrt{2g(H+h_2)}$$

if we take h_2 equal to the vertical distance of D′E′ above BC. We see thus that a diminution or augmentation of the pressure upon the free surface of the liquid in the vessel, without any change in that against the orifice at A, causes a corresponding diminution or augmentation in the velocity of the issuing fluid, and, on the contrary, that a diminution or augmentation of the pressure against the orifice, without any change in that upon the free surface, causes a corresponding augmentation or diminution in this velocity.

EXAMPLE.—The condenser of a low-pressure steam engine offers an example of the second case; for, let us suppose a vacuum of 25 inches of mercury to be maintained, and that the head of water in the cistern (Fig. 3) supplying the jet of cold water which effects the condensation, were 2 feet above the point at which it enters this partial vacuum, then the actual head producing the flow is $2 + 28.25 = 30.25$ feet, for pure mercury being 13.56 times heavier than water,—we have the height of a column of water which would balance that of 25 inches of mercury, equal to $25 \times 13.56 = 339$ inches, or 28.25 feet.

The self-acting contrivance for supplying the feed-water to low-pressure boilers (Fig. 4) comes under the first case: the pressure being supposed 5 lbs. per inch above the atmosphere, it is required to place the cistern of the supply so high, that on the opening of the valve a, by the float b descending below the proper level, the water may enter against the pressure of the steam. Now, as the cubic foot of water weighs 62.5 lbs., a column 1 foot high and 1 square inch base weigh $\frac{62.5}{144} = 0.434$ lbs., and, therefore, the height of the column of water to balance any given pressure expressed in pounds per square inch is found

by dividing that number by 0.434, in this case, $5 \div 0.434 = 11.52$ feet: this gives exact equilibrium; the additional head in order that it may enter with due rapidity (from 2 to 4 feet generally) will depend upon the consumption of the boiler and the area of the supply pipe. It is evident that this mode of supply is not convenient in high-pressure boilers; for suppose the pressure to be 50 lbs. per inch, then the height to produce equilibrium will be 115.2 feet. The pressure on an hydraulic ram is frequently 3 tons per inch, in pounds equal to (3×2240) 6720 lbs., and $6720 \div 0.434 = 15484$ feet.

If, instead of a free surface in the cistern, we had supposed a solid piston or plunger to press on the enclosed water, the head should in like manner be calculated by turning the pressure per square inch on the piston into vertical feet of water.

13. Having thus shown the law of the velocities of a fluid issuing from an orifice, let us proceed to apply it to the determination of its discharge, which is defined to be the volume of the fluid which escapes in the unit of time, i. e. one second.

If the mean velocity of all the particles was that due to the "charge" H, then this velocity, which is called the theoretic velocity, would be $\sqrt{2gH}$; and if at the same time the particles issued from all points of the orifice in parallel threads, it is evident that the volume of water flowing out in a second would be equal to the volume of a prism which would have the orifice for its base, and that velocity for height; and, calling S the area or section of the orifice, the volume of water, or of the prism, would be—

$$S \times V = S\sqrt{2gH}.$$

This is the theoretic discharge.

14. But the actual discharge is always less than this. In order to have an exact idea of the pheromena, let us consider the fluid vein a short distance after its issue from the orifice, and let us suppose it cut by a plane perpendicular to its direction. It is manifest that the discharge will be equal to the product of the section by the mean velocity of the several threads at the moment they intersect the plane of the section. If this section was equal to that of the orifice, and if this velocity was that due to the charge, then the actual discharge would also be equal to the theoretic discharge. But whether from the section of the vein being considerably less than that of the orifice,—as in the flow through orifices in a thin plate, or from the velocity being considerably less than that due to the charge, as in cylindrical adjutages; or, again, from a diminution in both the section and the velocity, as in certain

conical adjutages,—it always results, that the actual discharge is in every case less than the theoretic, and in order to reduce this last to the former, it is necessary to multiply it by some fraction. Let m represent this fraction, and Q the actual discharge, we shall then have—

$$Q = m\,S\sqrt{2gH},$$

and designating the volume of water flowing off in the time T by Q' we shall have—

$$Q' = m\,ST\sqrt{2gH};$$

whether the diminution in the discharge arises from a diminution in the section, or in the velocity, it is always a consequence of the contraction which the fluid vein suffers in passing through the orifice, and thus the multiplier m, or "coefficient for the reduction of the theoretic to the actual discharge," is generally called the "*coefficient of contraction.*" Its accurate determination is of the greatest importance; upon its degree of exactness depends that of the results we obtain when we would apply to practice formulæ upon the flow of water. We shall now proceed to give the results of experiments on the value of the symbol m, giving some preliminary statements, upon, first, the cause of the "contraction;" second, upon the nature of its effects; and third, upon the form of the fluid vein—the orifice being circular—its dimensions, and the effect of the form upon the discharge.

15. *Cause of the Contraction.*—If we take a glass vessel (Fig. 5), in the side of which is an orifice through which the water flows, and render visible the movement of the molecules of the water in the vessel by disseminating through it substances of equal specific gravity, and very minute, or by producing within the water some light chemical precipitation, such as occurs when we let fall a few drops of nitrate of silver in water slightly saline,—we then see at a small distance from the orifice—as, for instance, about an inch, when its diameter is three-eighths of an inch—the fluid molecules converge from all parts towards the orifice, describing curved lines, and finally, as if precipitated upon a centre of attraction, issue forth with a rapidly increased motion. The convergence of the directions that they had within the vessel at the moment of their arrival at the orifice, still continues for a short distance after they have passed out, so that we can plainly see the fluid vein after its passage gradually diminish, and become contracted up to the place where the particles, from the effect of their

mutual action, and of the motions impressed upon them, take directions, it may be parallel, or in some other lines. The vein thus forms a species of truncated pyramid or cone whose larger base is the orifice, and the smaller is the section of the fluid at its place of greatest contraction, a section which is often called the "section of contraction." This figure, and all the phenomena of contraction, are thus a consequence of the convergence of the several threads of water when they arrive at the orifice, or of the obliquity of their mutual directions.

16. *Effects of the Contraction.*—When the orifice is in a thin plate, the contraction is completely external to the reservoir; it is thus clearly visible, can be, and in fact it has been, measured, as we shall mention directly. When the orifice is circular, the fluid vein having reached the minimum section, continues of that transverse area, and is thus cylindrical in form, and has a velocity very nearly equal to that due to the charge. The discharge will therefore be the product of this section by the velocity, so that the effect of the contraction is limited to the reduction of the value of the section which enters into the expression of the discharge. The flow will take place as if the actual orifice had been replaced by another whose diameter was equal to the "section of contraction," but in which supposed orifice no true contraction took place.

If to the orifice AB (Fig. 6) we attach a tube or cylindrical adjutage, the fluid threads will arrive at AB, converging, and therefore the fluid will contract at the entrance. Experiments prove that this contraction is identical with that of the thin plate; it will, however, be internal with respect to the mouth of the tube. Moreover, beyond the "section of contraction" the attraction of the sides of the tube occasions a dilatation of the fluid vein; the threads follow these sides, and issue parallel to each other and to the axis of the tube: so that the section of the vein at its exit is fully equal to that of the orifice in the side of the reservoir, but the velocity is not that due to the charge. If the flow was solely produced by the pressure of the fluid, then the velocity at the section of contraction would be that due to the charge, and would diminish in proportion as the form of the fluid vein enlarged by virtue of that law or axiom of hydraulics—namely, when an incompressible fluid is in motion forming a continuous mass—then the velocity, at all its diverse sections, is inversely proportional to the area of the section: the diminution of velocity would then cease

when the fluid vein had, in diverging, reached the sides of the tube. Now since m is the ratio of the "section of contraction" to the section at the orifice, the velocity along the sides, and consequently at the mouth of the tube, would be $m\sqrt{2gH}$, and for the discharge we should have $S \times m\sqrt{2gH}$. In the orifices in a thin plate this discharge was $mS \times \sqrt{2gH}$, giving the same discharge in both cases,—the only difference being, that in the latter area is affected, in the former the velocity: in the case of the added cylindrical adjutages it falls on the velocity. But the attractive action of the sides of the adjutage alters this supposed state of things: not only does it cause the deviation of the fluid threads we have mentioned, it also augments their velocity so that the velocity of exit is greater than that given by the expression $m\sqrt{2gH}$; it will be $m'\sqrt{2gH}$, in which m' is greater than m, and the discharge that will be $S \times m'\sqrt{2gH}$.

We thus see, then, in cylindrical adjutages, and indeed in adjutages in general, the effect of the contraction of the fluid vein is complicated with that of the attraction of the sides. Without being able to assign that which belongs to the first alone, it may be said that every internal contraction is connected with the diminution of velocity, and that every external contraction produces a diminution of section.

17. *Form and Dimensions of the Contracted Vein of the Fluid.*—Let us next examine the form that it gives to the fluid vein issuing from an orifice, in the simple case of a circular orifice in a thin plate, truly plane. Everything being symmetrical around the different points of the orifice, the direction as well as the velocity of the molecules, the contracted vein ought also to be of a symmetrical form, and, consequently, a solid of revolution—a conoidal figure. It is actually so according to the observations that have been made, and which the figure AB *ab* (Fig. 7) represents. Beyond *ab* the contraction ceases, and the vein continues sensibly cylindrical for a certain length until the resistance of the air and other causes entirely destroy this form.

The earlier measurements that have been made give to the three principal dimensions AB, *ab*, and CD, the ratio of the numbers 1.00, 0.79, and 0.39. The length of the contracted vein is thus about half the diameter of the smaller section, and 0.39 of the larger, that is of the orifice.

18. Michelotti, from a mean of more recent experiments on a large scale, has adopted 1.00, 0.787, 0.498: these D'Aubuis-

son follows. The ratio of the diameters AB and ab being thus 1 to 0.787, that of the sections is 1 to $0.787^2 = 0.619$, that, namely, of the squares of the former numbers; thus, if s be the "contracted section," and S that of the orifice, we shall have—

$$s = 0{\cdot}619\ S,$$

and the discharge consequently, §§ 14 and 16—

$$s\sqrt{2gH}, \text{ or } 0.619\ S\sqrt{2gH},$$

so that the value of m, or the "coefficient of contraction," as determined by actual measurement, is at the mean equal to 0.619, and is a little less than that which results from experiments on the discharge. If the velocity at the passage of the "section of contraction" was actually that due to the charge, and that the flow took place through an adjutage of the exact form of the contracted vein, and that in the expression for the discharge the area s of the exterior orifice of this adjutage, taken at the extremity, were introduced, then the calculated would be equal to the actual discharge, and the coefficient of the reduction of the one to the other would be equal to unity; and Michelotti, in one of his experiments in which he employed a cycloidal adjutage, has reached 0.984. It is very probable he would have actually reached 1 if this form had more accurately been adapted to that of the fluid vein, and if the resistance of the air had not somewhat retarded the motion.

19. *Flow of Water through an Orifice in a Thin Plate.*—We come now to the direct determination of the co-efficient for reducing the theoretical to the actual discharge. For this purpose it is necessary to gauge with care the volume of water discharged in a given time under a constant charge, from which we deduce the flow in one second, or the actual discharge; and dividing this by the theoretic discharge for the same head and same orifice, the quotient is the *coefficient* required. EXAMPLE.—Thus, with a head of 4 feet we have a velocity of 16.07 feet per second; and the diameter of the orifice being 3 inches, we have its area equal to $3^2 \times 0.7854 = 7{\cdot}97$ square inches, and $\frac{7{\cdot}97}{144} = 0.0554$ its value in square feet; this, multiplied into the velocity of the water, gives the volume of the prism or cylinder equal to that of the water discharged; that is (§ 13), $0.0554 \times 16.07 = 0.8903$ cubic feet per second. But having found that in $1\frac{1}{2}$ minutes the

actual discharge is 49.68 cubic feet, and reducing this to its value for 1 sec. by dividing by 90, we obtain $\frac{49.68}{90} = 0.552$ cubic feet as the discharge in 1 sec.; hence, dividing the actual by the theoretic discharge, we find for the *coefficient* $\frac{0.552}{0.8903} = 0.620$. Very many hydraulicians have for a long time been engaged in its determination. The following Table, from D'Aubuisson, gives the principal results obtained by experiments up to the present time, and which, having been made under favourable circumstances, are generally received. They include circular, square, and rectangular orifices:—

CIRCULAR ORIFICES.			
Observers.	Diameters.	Charges.	Coefficient.
	Feet.	Feet.	
Mariotte, .	0.0223	5.8712	0.692
Do.	0.0223	25.9120	0.692
Castel, . .	0.0328	2.1320	0.673
Do.	0.0328	1.0168	0.654
Do.	0.0492	0.4526	0.632
Do.	0.0492	0.9840	0.617
Eytelwein, .	0.0856	2.3714	0.618
Bossut, . .	0.0889	4.2640	0.619
Michelotti, .	0.0889	7.3144	0.618
Castel, . .	0.0984	0.5510	0.629
Venturi, . .	0.1345	2.8864	0.622
Bossut, . .	0.1771	12.4968	0.618
Michelotti, .	0.1771	7.2160	0.607
Do.	0.2657	7.3472	0.613
Do.	0.2657	12.4968	0.612
Do.	0.2657	22.1728	0.597 ?
Do.	0.5314	6.9208	0.619
Do.	0.5314	12.0048	0.619

SQUARE ORIFICES.			
Observers.	Side of Square.	Charges.	Coefficient.
	Feet.	Feet.	
Castel, . .	0.0032	0.1640	0.655
Bossut, . .	0.0885	12.5000	0.616
Michelotti, .	0.0885	12.5000	0.607
Do.	0.0885	22.4078	0.606
Bossut, . .	0.1771	12.5000	0.618
Michelotti, .	0.1771	7.3472	0.603
Do.	0.1771	12.5624	0.603
Do.	0.1771	22.2384	0.602
Do.	0.2689	7.3489	0.616
Do.	0.2656	12.5624	0.619
Do.	0.2656	22.3700	0.616

RECTANGULAR ORIFICES.			
Rectangle.		Charges.	Coefficients.
Height	Base.		
Feet.	Feet.	Feet.	
0.0301	0.0606	1.0824	0.620
0.0301	0.1213	1.0824	0.620
0.0301	0.2423	1.0824	0.621
0.0301	0.4847	1.0824	0.626

20. The experiments of Michelotti were carried on about three miles from Turin, at an hydraulic establishment constructed for experimental purposes, consisting of a building 26 feet high, supplied with water from the River Dora by a canal of derivation. The internal dimension was a square of 3 feet 2¼ inches; on one of the sides was arranged a series of adjutages at the different depths deemed expedient, and upon the surface of the ground were arranged the different receptacles for the gauging of the actual discharges. It may be remarked upon this part of the Table, that the coefficients obtained from the large orifices are higher than the others, and this contrary to the rule that would be deduced from the experiments in general.

21. In order to place the subject of the variation in the value of the coefficient, under different circumstances of area and charge, in a clear point of view, the following Table of MM. Poncelet and Lesbros' experiments at Metz in 1826 and 1827 is given. In these experiments the orifices were rectan-

gular, and all of the same breadth—namely, $0^{m}.20 = 0.656$ ft.; the heights were successively 0.656, 0.328, 0.164, 0.098, 0.065, and 0.0328 feet. The charges extended from 0.33 feet to 5.58 feet. With each orifice they repeated the experiments, and took them with 8 or 10 charges from the smallest to the highest that the apparatus admitted, and the corresponding coefficients were calculated. They then took the charges for the abscissæ, and these coefficients for the ordinates of a curve constructed for each orifice, and by its aid they determined the ordinates,—that is, the coefficients intermediate to those directly determined by experiment; and thus gave a very extended Table, from which the following is taken:—

Charge on Centre of Orifice.	HEIGHT OF THE ORIFICES.						Difference of maximum and minimum coefficients.
	Feet. 0.656	Feet. 0.328	Feet. 0.164	Feet. 0.098	Feet. 0.065	Feet. 0.032	
Feet.							
0.032						0.709*	
0.065					0.660	0.698	
0.098				0.638	0.660*	0.691	
0.131			0.612	0.640	0.659	0.685	
0.164			0.617	0.640	0.659	0.682	
0.196		0.590	0.622	0.640*	0.658	0.678	
0.262		0.600	0.626	0.639	0.657	0.671	
0.328		0.605	0.628	0.638	0.655	0.667	
0.393	0.572	0.609	0.630	0.637	0.654	0.664	0.092
0.492	0.585	0.611	0.631	0.635	0.653	0.660	0.075
0.656	0.592	0.613	0.634*	0.634	0.650	0.655	0.063
0.984	0.598	0.616	0.632	0.632	0.645	0.650	0.052
1.312	0.600	0.617*	0.631	0.631	0.642	0.647	0.047
1.640	0.602	0.617	0.631	0.630	0.640	0.643	0.041
2.296	0.604	0.616	0.629	0.629	0.637	0.638	0.034
3.281	0.605*	0.615	0.627	0.627	0.632	0.627	0.027
4.264	0.604	0.613	0.623	0.623	0.625	0.621	0.021
5.248	0.602	0.611	0.619	0.619	0.618	0.616	0.017
6.562	0.601	0.607	0.613	0.613	0.613	0.613	0.012
9.843	0.601	0.603	0.606	0.607	0.608	0.609	0.008

Fig. 8 illustrates this method. From the point O the several charges are laid off on the line ON, as OX, OX_1, &c., and the corresponding coefficients XY, X_1Y_1, &c.; and the curve being traced through Y, Y_1, Y_2, &c., we can obtain the coefficient proper to any charge Ox by drawing the perpendicular xy terminating in the curve.

22. All the numbers contained in this Table are the several values of the coefficient m in the formula $Q = mS\sqrt{2gH}$. But those in each column above the transverse line are not the true

coefficients for the reduction of the theoretic to the actual value, as will be shown hereafter. Casting the eye over each column, we may see that the coefficients increase as the charges are greater, but up to a certain point only, although the charge still increases: an asterisk in each column points out the respective maxima. It may also be observed, that the coefficients approach equality in each column as the charges increase,—the bottom line of figures, in which the charge was $3^{m} = 9.84$ feet, being almost identical in each column.

23. This Table, although constructed from experiments on rectangular orifices, can yet be extended to those of all other forms,—the height of the rectangle, as given in the Table, corresponding to the smaller dimension of the orifice made use of; for we find it generally admitted that the discharge is altogether independent of the figure of the orifice when the area is constant, provided only that this figure has no re-entrant angles.

24. Although these experiments of MM. Poncelet and Lesbros are on a considerable scale, yet there are some cases in actual practice in which the discharge is twenty or thirty times greater. Such are the sluices in lock-gates on canals of navigation; and it is a matter of some importance to determine directly the coefficient of discharge for them. The following Table gives the result of experiments on the canal of Languedoc; the width of the sluice was $1^{m}\ 30 = 4.25$ feet nearly:

Sluice.		Charge upon the Centre.	Discharge in 1 sec.	Coefficient.
Area.	Height.			
Square Feet.	Feet.	Feet.	Cubic Feet.	
7.7442	1.804	14.550	145.3056	0.613
6.9928	1.640	6.628	92.6438	0.641
6.9928	1.640	6.245	88.2288	0.629
6.4664	1.508	12.874	138.6302	0.641
6.7237	1.574	12.582	128.7759	0.647
6.7237	1.574	6.392	83.9551	0.616
6.7237	1.574	6.215	79.8580	0.594
6.7172	1.574	6.478	85.2266	0.621
Mean Coefficient				0.625

This mean coefficient is rather greater than that found by Poncelet (§ 21), which is readily explained, as the flow of water did not take place as if in a thin plate, the contraction being suppressed on some parts of the boundary. The wood

work which surrounded the sluiceway was = 0.8856 feet thick, and on the sill was even = 1.771 ft. Thus, when the sluice was raised but a small height, the contraction ceased on four sides, and the coefficient was considerably increased. For example, when the paddle was raised only 0.393 ft., it gave a coefficient of 0 803; when raised 1.51 ft., it was 0.641.

25. *Particular Cases in which the Contraction is suppressed on one or more sides of the Orifice.*—In all the different cases treated of hitherto, it has been assumed that the fluid arrived at the orifice from all parts equally, but frequently this is not so. For example (Fig. 9), when the orifice is at the bottom of a vertical plate, and that its inferior edge is on the level of the bottom of the vessel or reservoir, the contraction is then destroyed on that side, and, consequently, the discharge is increased. The question arises, therefore, how much will the discharge be augmented by the suppression of the contraction for a certain length of the periphery of the orifice? The following Table gives the result of experiments instituted with the view of determining this point. The orifice was rectangular, 0.177 feet in base, and 0.089 feet in height. The plates, which were attached, sometimes on one side, sometimes on two or three of the sides, were 0.22 feet long; that is, they advanced this much into the reservoir. The flow was produced by charges from 6.56 feet to 22.56 feet in height:—

Portion of Orifice without Contraction.	Coefficient.	Ratio of Increase.
0	0.608	1.000
$\frac{1}{6}$	0.620	1 020
$\frac{2}{6}$	0.637	1.049
$\frac{3}{6}$	0 659	1.085
$\frac{4}{6}$	0.680	1.119
$\frac{5}{6}$	0.692	1.139

26. In this Table the last column has for its unit the discharge when the orifice is perfectly free: the numbers, therefore, indicate the increase in the coefficients, and consequently in the discharges. The formula deduced by M. Bidone, the experimenter, is $1 + 0.152 \frac{n}{p}$, in which n represents the length of the part of the perimeter in which the contraction is suppressed, and p the perimeter of the orifice. The greatest error of this formula being but the $\frac{1}{30}$th part, it may be used for the value of the discharge when, in the case of rectangular ori-

fices, there is no contraction on part of the boundary, and the actual discharge then is $m\ S\ \sqrt{2gH}\ (1 + 0.152\ \frac{n}{p})$.

27. *Orifices in plates not being true planes.*—The supposition hitherto has always been that the sides or plates in which the orifices were placed were truly plane; they may, notwithstanding, be of surfaces very different. In order to have a clear idea of the effect which this alteration produces upon the flow, it is necessary to recall to mind that if the threads of the fluid vein did arrive at the orifice mutually parallel, the actual discharge would be equal to the theoretic, and that it is less only by reason of the oblique directions in which they meet, from which necessarily results a destruction of part of the acquired motion at the point of contact. If, therefore, we imagine around the orifice a spherical surface of a radius equal to that of the sphere of action of the orifice, and this surface terminated by the sides of the vessel, then it must be intersected on every point, and in direction nearly perpendicular, by the threads of the fluid (Figs. 10 and 11); and the larger the part of the sphere this surface may be, and the more oblique, or even opposite, to one another, the threads of the fluid arrive at it, then the more the motion is destroyed at the entrance of the orifice, and the less the discharge is found to be. When the sides are developed in one plane, then this supposed surface is a hemisphere (Fig. 6), and the coefficient of the particular case is given above, p. 15, § 21. But if they are disposed in the form of a funnel, or, if simply concave, in the interior of the vessel, then the surface of this sphere is of less extent and the discharge more considerable,—not, however, that it follows the exact proportion of the spherical surface If, on the other hand, the side is convex, the discharge is diminished, and it will be less still in the case represented in Fig. 10. Lastly, it will be at its minimum if the supposed surface should become an entire sphere; and this would happen if it was possible to carry an orifice into the midst of the mass of the fluid enclosed in the vessel.

28. Borda has succeeded in realizing this case almost completely. He has introduced into a vessel (Fig. 11) a tube of tin 0.443 feet long and 0.105 feet in diameter, and under a charge of 0.82 feet he has caused the flow to take place, so that the effluent water did not touch the sides of the tube at all. The actual discharge has been only 0.515 of the theoretic, and from various circumstances Borda was led to think that he might have reduced it to 0.50.

Having subsequently surrounded the orifice of the entry

of the tube with a border or rim, and having thus reduced it to the condition of being in a perfectly plane plate, although in the centre of the fluid mass, he found the coefficient rise to 0.626. The same result might be obtained by employing a simple tube, but formed of a thick material. Fig. 12 shows the manner in which the fluid bends around the exterior edge, and enters the tube without touching the internal sides,—the thickness being about $\frac{1}{12}$th of an inch, or 0.0065 feet, and the edges cut truly square: thus all that part of the sides within the exterior periphery is, as far as the discharge is concerned, as if totally removed; and it is this external diameter that should be introduced in all calculations relative to internal adjutages. By taking it M. Bidone has found, from two experiments in which the effluent fluid did not touch the sides, that the coefficient was nearly 0.50,—that is, the section of contraction was half the orifice taken at the external circumference.

29. Thus 0.50 and 1.00 will express the limits of the coefficients of contraction,—limits to which they may approach very nearly, but which they can never actually attain. For orifices in a plate truly plane it does not descend below 0.60, or rise much above 0.70. and in ordinary practice it ranges between 0.60 and 0.64. As a mean term, 0.62 is generally taken; so that—

$$Q = m \quad S\sqrt{2gH} = 0.62 \quad S\sqrt{2gH} = 4\,96\ S\sqrt{H};$$

and if the orifice be circular of a diameter d, the area is expressed by $d^2 \times 0.7854 = S$ and $Q = 3.9\,d^2\sqrt{H}$. For greater exactness in the coefficient, recourse should, however, be had to the Table, page 15, § 21.

30. But with respect to the velocity of the effluent fluid in orifices in a thin plate truly plane, is it, as we have assumed, actually equal to that due to the charge—is it, in fact, $\sqrt{2gH}$ or $8\sqrt{H}$, in feet per second? We may deduce this velocity from observing the height to which the water rises in a vertical jet, as in § 8. Another method, however, of determining the measured velocity enables us to estimate it still more accurately. To have a clear notion of this method, it is necessary to recall the following principles:—When a body is projected in any direction AY (Fig. 13) with a certain velocity, the combined action of this velocity with the force of gravity causes it to describe a curved path, AMB. If the velocity, and consequently the resistance of the air, be not very great, the curve is a parabola. The demonstration of this may well be repeated here though given in many works.

Let v (Fig. 13) be the velocity with which the body is sent forth in the direction of AY, and t the time spent in reaching the point N, then, since the velocity in the direction AN is uniform, AN $= vt$; on the other hand, if the body had been only under the action of the accelerating force of gravity, it would have descended from A to some point P during that same interval t, so that we should have AP $= \frac{1}{2}gt^2$. If we complete the parallelogram APMN, the point M will have been reached under the joint action of these movements in the same time, t, in which the point P was attained under the sole accelerating force; and it will have, therefore, traversed the arc of the curve, whose abscissa will be AP, and ordinate MP, parallel to the axis AY. Let x = AP and y = MP, we have therefore

$$(a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad x = \frac{gt^2}{2} \text{ and}$$

$$(b) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad y = vt.$$

From (b) we have $t = \frac{y}{v}$ and $t^2 = \frac{y^2}{v^2}$, and substituting this value of t^2 in (a), we have $\frac{gy^2}{2v^2} = x$, or $y^2 = \frac{2v^2x}{g} = \frac{4v^2x}{2g}$, or calling h the height due to the velocity v, and remembering that $\frac{v^2}{2g} = h$, we have

$$(c) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad y^2 = 4hx,$$

which is the well-known equation of the parabola, of which $4h$ is the parameter. Hence this theorem, "that a heavy body projected with any force whatsoever describes a parabola, whose parameter is equal to four times the height due to the velocity of projection."

31. This truth, which has been proved for any body in general, holds good also for a jet of water issuing from an orifice (Fig. 14). If this orifice be opened in a vertical plate, the axis of projection being horizontal, the ordinates will be horizontal,—that is, the distances of the different points of the jet from the vertical let down from the centre of the orifice; and if through any point C of this vertical we draw a horizontal plane, then, according to the theorem, the square of the distance CD—called the range of the jet—taken in this plane (or generally of the distance MP), divided by four times the corresponding fall, AP, will give the height due to the velocity of exit $h = \frac{y^2}{4x}$. Thus the permanent form that the jet of water assumes being

identical with the path of any single particle acted on by the same forces, we are enabled to use it as a mode of measuring the velocity of the water at its issue from the orifice. A vertical rod divided into any scale of equal parts, and firmly fixed, having its zero at the centre of the orifice, has applied to it at right angles another rod similarly divided, and having a stock like a T square, so as to slide up or down the vertical fixed rod, and its zero being in the vertical let fall from the mouth of the orifice, we can then measure any ordinate PM and corresponding abscissa AP. Hence, by measuring y and x, we can calculate h, the height due to the velocity of exit, from the formula just given. Dividing both sides of equation (c) by $4x$ we have $h = \frac{y^2}{4x}$, and comparing h so found with H, the charge, we find them very nearly coincident, as in the following Table of experiments by Bossut:—

Curve of the Jet. Abscissa = x	Curve of the Jet. Ordinate = y	Height due to Velocity of Exit.	Charge.	Differences.
Feet.	Feet.	Feet.	Feet.	Feet.
20.598	24.698	7.404	7.511	0.107
15.284	27.716	12.564	12.890	0.326
4.624	20.500	22.720	23.583	0.863

Thus $\frac{24.668^2}{4 \times 20.598} \; \frac{610.1}{84.2} = 7.404$, and so of the other numbers in the third column.

The difference between the third and fourth columns increases with the charge, and we should expect it to be so, since the cause of this difference—the resistance of the air—increases as the square of the velocity, and, consequently, nearly as the charge. Had it not been for this, the difference had been very nearly equal to zero, and the velocity at the section of contraction, as mentioned in p. 4, § 8, is truly stated as equal to that due to the charge. This general proposition may consequently be laid down:—"Water flowing through an orifice in a vertical thin plate issues with a velocity, q. p., equal to that due to the charge, and is not sensibly diminished by the *venâ contracta.*"

32. *Effects on the Discharge when the Fluid has Velocity antecedently.*—If the water contained in the reservoir, instead of being in a state of repose, was moving in the direction of the orifice,—as when the vessel, having but a relatively small

section, has a supply of water brought into it, and flowing directly up to the plate or side in which the orifice is opened,—then the particles of the fluid would issue not only in virtue of the pressure exerted by the fluid mass which is above it, but with the additional velocity that they had when they entered into the sphere of action of the orifice; we must therefore add to the actual charge measuring the pressure a new term, which will be the height due to this supposed velocity of arrival. Thus, if u represent this velocity, we shall have (since $\frac{u^2}{2g}$ is the height producing the velocity u) the expression—

$$Q = mS\sqrt{2g\left(h + \frac{u^2}{2g}\right)} = mS\sqrt{2gh + u^2}.$$

33. For example:—A basin 20 mètres long, and 2^m wide, and 1^m deep, has at one extremity a plank in which is cut a rectangular orifice $0^m.55$ wide, and $0^m.36$ deep; its sill or lower edge is $0^m.91$ below the constant level at which the water in the basin remains. At the other end it receives a supply of water: what then is the discharge?

$$S = 0.55 \times 0.36 = 0^{mm}.198$$
$$h = 0.91 - \tfrac{1}{2}\,0.36 = 0^m.73$$
$$m = 0.600$$

the value of m being deduced by extending the Table, § 21.

The value of Q, taken first in neglecting u, will be—

$$0.600 \times 0.198\sqrt{2g \times 0.73} = 0^{mm}.4496 = 15.878.$$

Now, when water flows in a canal with a mean velocity u, we evidently have, s being the transverse section, $Q = su$. Then dividing by s, we have $\frac{Q}{s} = u$, or as $s = 2^m$

$$\frac{0.4496}{2} = 0^m.2248 = u,$$

and, therefore,

$$u^2 = 0.0505.$$

And putting this value in the general expression—

$$Q = 0.6 \times 0.198\sqrt{14.321 + 0.0505} = 0^{mmm}.45025 = 15.9 \text{ cb. ft.}$$

The French measures have been here retained, to afford an example for the reduction of the mètre into English feet, 1 mètre being 3.281 ft.

34. *Flow of water with cylindrical adjutages.*—We have already seen (§ 16) that the addition of a tube gives a discharge larger than that through an orifice in a thin plate; but in order that it should produce this effect it is necessary that the water entirely fill the area of the external mouth of the tube, and this is generally the case when the length of the tube is three or four times greater than its diameter: if it be less than this, it frequently occurs that the fluid vein which is contracted at the entrance does not enlarge so as to fill the interior of the tube; the flow in that case takes place as if in a thin plate, and this is always the case when the length of the tube is less than the length of the contracted vein, and therefore but half the diameter, or less. The coefficient of the reduction of the theoretic to the actual discharge is given in the following Table:—

Observers.	Adjutage.		Charge.	Coefficient.
	Diameter.	Length.		
	Feet.	Feet.	Feet.	
Castel, .	0.0508	0.1312	0.656	0.827
Do.	0.0508	0.1312	1.574	0.829
Do.	0.0508	0.1312	3.247	0.829
Do.	0.0508	0.1312	6.560	0.829
Do.	0.0508	0.1312	9.938	0.830
Bossut, .	0.0754	0.1771	2.132	0.788
Do.	0.0754	0.1771	4.067	0.787
Eytelwein,	0.0852	0.2558	2.361	0.821
Bossut, .	0.0885	0.1344	12.628	0.804
Do.	0.0885	0.1771	12.693	0.804
Do.	0.0885	0.3542	12.857	0.804
Venturi, .	0.1344	0.4198	2.886	0.822
Michelotti,	0.2656	0.7084	7.150	0.815
	Square.			
Do.	0.2656	0.7084	12.464	0.803
Do.	0.2656	0.7084	22.008	0.803

The mean of these coefficients gives 0.817, and it is generally taken as 0.82, so that we have the following formulæ:—

$$Q = 0.82 \times S \sqrt{2gH} = 6.56 \quad S \sqrt{H} = 5.152\, d^2 \sqrt{H}.$$

35. In the case when the jet issues with the tube full, in threads parallel to the axis of the orifice, and when, conse-

quently, the section is equal to that of the orifice, the diminution of the discharge can only occur from a diminution of the velocity; and the ratio of the actual to the theoretic discharge is the same as that of the actual to the real velocity. The Table below, giving two experiments by Castel, and a third by Venturi, proves this:—

Observers.	Coefficient of the	
	Velocity.	Discharge.
Venturi,	0.824	0.822
Castel, .	0.832	0.827
Castel, .	0.832	0.829

Showing that the coefficients of velocity and discharge are sensibly equal. The three quantities measured were the "charge" on the centre of the tube, the velocity computed as in § 31, and the volume discharged. The velocity due to the charge, compared with that so computed, gives the second column, and the product of the area of the tube into the velocity due to the charge, compared with the discharge, gives the third; that is—

$$\underline{V}$$
$$\sqrt{2gH} : \text{computed Velocity}, \quad :: 1 : 0.824, \text{ and}$$
$$S \times V : \text{Discharge} \quad . \quad . \quad . \quad :: 1 : 0.822.$$

We must therefore conclude that the velocity of a jet of water at the extremity of a cylindrical adjutage is equal to 0.82 of that due to the charge, and that the height due to that velocity is but 0.67 of the actual height of the reservoir; that is $(0.82)^2$, because the heights or charges are as the squares of the velocities.

36. As to the cause of this increase of the coefficient from 0.62 to 0.82, D'Aubuisson ascribes it to the attraction of the sides of the tube and the divergence of the fluid threads. After they have come in contact with them, they are forcibly retained by the molecular attraction, such as that which causes the rise of fluids in capillary tubes: by this same force the outer threads draw after them the inner, and so all the vein issues with a full tube, and passes with a greater velocity through the contracted section. The immediate cause is in the contact, and every circumstance which favours that tends to produce an augmentation of the coefficient.

37. *Flow of water through conical converging adjutages.*—Conical adjutages, properly so called,—that is, those which are slightly converging to a point exterior to the reservoir, augment the discharge still more than the preceding. They give jets of great regularity, and throw the water to a greater distance or height, and are hence frequently used in practice: the effects vary with the angle of convergence of the sides. Two distinct contractions of the fluid vein take place with this adjutage—one interiorly, or at the entrance of the adjutage, which diminishes the velocity due to the charge; the other at the exterior; in consequence of which the true section of the fluid vein is slightly less than the area of the external mouth of the adjutage. If, therefore, we put S for the section of the orifice, and V for the velocity due to the charge, the actual discharge will be expressed by $nS \times n'V = nn'SV$, the two coefficients n and n' must be found by experiment, n being the ratio of the fluid section to that of the orifice, or the coefficient of the exterior contraction, and n' that of the actual velocity to the theoretic, or the *coefficient* of *the velocity*, and nn' their product, is the ratio of the actual discharge to the theoretic, or the *coefficient* of *the discharge*. The knowledge of these two last is of some importance in the case of jets of water, as in fountains and fire-engines.

38. In order to determine the different coefficients mentioned, and especially to fix the angle of convergence that would give the maximum discharge, experiments were undertaken with a number of adjutages successively, in each of which the diameter of the orifice of final issue and the length of the adjutage remained constant; but in each the diameter of entrance, and consequently the angle of convergence, was increased. The flow of the water was produced with different charges with each of these varied adjutages. At every experiment the discharge was determined by actual gauging, and the velocity of issue by the method of the parabola, given above (§ 31). The discharge, divided by SV, gave the product nn', and the observed velocity divided by V $(= \sqrt{2gH})$ gave n'. The series of the numbers nn' showed the discharge corresponding to each angle of convergence, and consequently the angle of maximum discharge, and the series of n', marked the progression by which the velocities increased.

39. The same adjutage, under charges which varied from 0.69 feet to 9.94 feet, always gave discharges proportional to $\sqrt{H}$, and therefore the coefficient has been, q. p., the same also. A very small increase may be observed with the higher

charges. The Table gives those—for each series of diameters, $0^m.0155$ and $0^m.020$ = 0.051 feet to 0.066 feet—obtained by the adjutages of the maximum discharge.

Charge.	Coefficient of the	
	Discharge = nn'.	Velocity = n'.
Adjutage, . . . 0.0508 feet diameter.		
Feet.		
0.705	0.946	0.963
1.584	0.946	0.966
3.253	0.946	0.963
4.893	0.947	0.966
6.579	0.946	0.956
9.938	0.947	
Adjutage, . . . 0.0656 feet diameter.		
0.692	0.956	0.966
1.584	0.957	0.968
3.263	0.955	0.965
4.913	0.956	0.962
6.586	0.956	0.959
9.938	0.957	

With respect to the coefficients of the velocity, they also should have been found constant but for the resistance of the air. Now this resistance diminishing the throw of the jet, and that in proportion as the charge is greater, we should expect in the coefficients calculated from it, a decrease augmenting with the charge,—although, at the same time, there was no actual diminution in the velocity with which the fluid issued, or tended to issue. Let us, in the next place, compare together the coefficients both of the discharges and of the velocities obtained with the different adjutages of one and the same series,—adjutages which only differed in the angle of their convergence. For each coefficient a mean of five or six has been taken, of those given by several different charges, very nearly the same as those put down in the preceding Table.

Angle of Convergence.	Coefficient of the Discharge.	Coefficient of the Velocity.	Angle of Convergence.	Coefficient of the Discharge.	Coefficient of the Velocity.
Diameter, . . 0.0508 feet.			Diameter, . . 0.0656 feet.		
0° 0′	0.829	0.830			
1° 36′	0.866	0.866			
3° 10′	0.895	0.894	2° 50′	0.914	0.906
4° 10′	0.912	0.910			
5° 26′	0.924	0.920	5° 26′	0.930	0.928
7° 52′	0.929	0.931	6° 54′	0.938	0.938
8° 58′	0.934	0.942			
10° 20′	0.938	0.950	10° 30′	0.945	0.953
12° 4′	0.942	0.955	12° 10′	0.949	0.957
13° 24′	0.946	0.962	13° 40′	0.956	0.964
14° 28′	0.941	0.966	15° 2′	0.949	0.967
16° 36′	0.938	0.971			
19° 28′	0.924	0.970	18° 10′	0.939	0.970
21° 0′	0.918	0.971			
23° 0′	0.913	0.974	23° 4′	0.930	0.973
29° 58′	0.896	0.975	33° 52′	0.920	0.979
40° 20′	0.869	0.980			
48° 50′	0.847	0.984			

40. It follows, from the facts pointed out in this Table, 1st, That, for the same orifice of issue, and with the same constant charge, the actual discharge, commencing with a discharge equal to 0.83 of the theoretical, increases gradually in proportion as the angle of convergence increases; but only up to $13\frac{1}{2}°$, at which the coefficient of discharge is 0.95: beyond this angle it diminishes at first slowly, as do all variables, about the maximum. At 20° the coefficient is yet as high as 0.92 to 0.93; subsequently the diminution becomes more and more rapid, and terminates as low as 0.65, which is the coefficient of discharge through a thin plate,—this last being the ultimate position of converging adjutages, that, namely, in which the angle of convergence has attained its maximum, or 180°. Thus, then, we have for the maximum discharge an angle of convergence of between 13° and 14°.

2ndly. In looking down the column of coefficients for the velocity, we see them also increasing from the angle 0° as do those of the discharge, and up to 10°; after that they increase more rapidly; and when beyond the angle of maximum discharge, while this last diminishes, they continue to augment

and approach their limit of unity. They are very nearly equal to 1 at 50°, and even at 40° are not far from it. Conical adjutages may, by varying the angle of convergence, be made to form, as it were, a series or progression, whose first term is the cylindrical adjutage, and last the orifice in thin plate; the velocity of projection, then, increasing with the angle of convergence, will vary from that of the tube additional, up to that of the simple orifice in a thin plate; that is to say, from $0.82 \times \sqrt{2gH}$ up to $1 \times \sqrt{2gH}$.

3rdly. If we compare the coefficients of discharge with those of the velocity,—that is, the successive values of $n \times n'$ and n', and dividing the former by the latter,—we shall have the values of n, or the coefficients of the exterior contraction. From the angle 0° up to 10°, n is sensibly equal to 1, and, consequently, no such contraction was present in the experiment; and notwithstanding the convergence of the sides, the fluid particles issued, q. p., parallel to the axis of the cone. Beyond 10°, however, the contraction becomes apparent; it reduces more and more the section of the vein, and terminates by bringing it to an equality with that of the orifice in the thin plate, as is shown here:—

Angle.	n
8°	1.00
15°	0.98
20°	0.95
30°	0.92
40°	0.89
50°	0.85
180°	0.65

In these experiments the length of the conical adjutage was fixed at about $2\frac{1}{2}$ times the diameter, measured at the exterior: thus it was 0.1312 feet for those of 0 0508 feet, and 0.1640 feet for those of 0.0656 feet, in order, as far as possible, not to complicate the results with the friction against the sides, and in this following the analogy of the cylindrical adjutage, in which experience proves that with respect to discharge they produce their full effects most certainly when their length equals $2\frac{1}{2}$ times their diameter.

41. As to those very large conical adjutages, or rather truncated pyramidal tubes, which in manufactories discharge water upon mill-wheels, three very valuable experiments, made at a mill on the canal of Languedoc, are given by the engineer,

Lespinasse. They were, in this case, formed by the sides of a rectangular pyramid, whose length was 9.59 feet; rectangle of large end, 2.4 feet by 3.2 feet; at the smaller, 0.443 feet by 0.623 feet.

The opposite faces made angles of 11° 38′ and 15° 18′; the charge was 9.6 feet:—

Discharge.	Coefficient.
Cubic Feet.	
6.767	0.987
6.692	0.976
6.714	0.979

We see, then, how very little such adjutages diminish the discharge: that which they give is only one or two hundredths below the theoretic discharge.

42. *Conical adjutages diverging.* — This adjutage, of all others, gives the largest discharge. It may be described as a truncated cone attached to a reservoir by its smaller diameter, and of which the exterior mouth is consequently greater than that of the entry of the water. Although not much in practical use, they present phenomena of such interest as to deserve some notice.

The property they have of increasing the discharge was known to the ancient Romans: some of the citizens, to whom had been granted the privilege of having a certain quantity of water from the public reservoirs, found, by using these adjutages, the means of increasing their supply; and the fraud became so extensive that their use was forbidden by law, except when the distance from the reservoir was not less than about 52 feet. Venturi is the experimenter to whom we are chiefly indebted for information respecting this particular adjutage.

43. Those which he made use of carried a mouthpiece, ABCD, not unlike the form of the contracted vein (Fig. 13), AB being equal to 0.1332 feet, and CD equal to 0.1109 feet. The body of the adjutage varied in length and in its expansion: this last was measured by the angle contained between the sides EG and FD, supposed prolonged until they meet. These adjutages were adapted to a reservoir maintained at a constant level; the flow took place under a constant charge of 2.89 feet; and the time required to fill a vessel of 4.838 cubic feet was observed.

The following Table gives the result of his principal observations, premising that the time corresponding to the theoretic velocity was 25.49 seconds:—

Adjutage.		Time of Flowing.	Coefficient.
Angle.	Length.		
	Feet.	Seconds.	
3° 30′	0.364	27.5	0.93
4° 38′	1.095	21	1.21
4° 38′	1.508	21	1.21
4° 38′	1.508	19	1.34
5° 44′	0.577	25	1.02
5° 44′	0.193	31	0.82
10° 16′	0.865	28	0.91
10° 16′	0.147	28	0.91
14° 14′	0.147	43	0.61

Venturi has drawn the conclusion that the adjutages of maximum discharge should have a length of nine times the diameter of the smaller base, and an angle of divergence equal to 5° 6′: it is represented in Fig. 13. This, he adds, would give a discharge 2.4 times greater than the orifice in a thin plate, and 1.46 times greater than the theoretic discharge.

44. *Flow of water under very small charges.*—When the charge over the centre of the orifice is very small compared with the vertical depth of the orifice itself, then the mean velocity of the different threads of the fluid vein—that is to say, the velocity which, being multiplied by the area of the orifice, gives the actual discharge—is no longer that of the central thread. It differs from it in proportion as the charge is less: its true value will be about the hundredth part less when the charge is equal to the depth of the orifice, and by about the thousandth part when equal to three times that depth. Let us examine what theory teaches under this head: and first, of the law which it assigns for the velocity of the fluid threads in proportion as their depth below the surface of the water in the reservoir at which they issue increases.

45. *Velocity of any fluid thread whatever.*—Let A (Fig. 16) be the level of the surface of water in a vessel, and upon the face AB—which, for greater simplicity, we suppose vertical—let us imagine a series of small orifices placed one below the other, and of which that at B is the lowest, and putting H for the height AB, the velocity of the jet issuing from B will

be expressed by $\sqrt{2gH}$; if we make BC equal to this quantity, it will represent this velocity; for any other point, P, taken below the surface of the water at a depth equal to AP or x, the velocity of the issue will be represented by the line PM $= \sqrt{2gx}$, and calling this y, we shall have—

$$y = \sqrt{2gx}.$$

If through all the points M so found we draw a curve-line they will be the ordinates, and the heights AP or x will be the abscissæ; and because $y^2 = 2gx$, this curve will be a parabola, having $2g$ or 64.4 feet for its parameter, and thus we have this proposition:—The velocity of a fluid thread issuing from a reservoir at any point is equal to the ordinate of a parabola whose parameter is equal to twice the measure of the force of gravity, and the depth below the surface of the water is the abscissa.

46. *Volume discharged.*—Let us, in the next place, suppose that instead of this series of small orifices in the face AB, we had opened a rectangular slit, whose width was l, and inquire the discharge in this case,—suppose this opening divided into a number of elementary rectangles by horizontal lines very close to each other, the volume of water which will issue from each of them in one second, or its discharge, will evidently be equal to the volume of a prism, whose base is the elementary rectangle, and height the velocity of issue; i. e. the ordinate corresponding. The sum of all the small prisms, or the total discharge, will also be equal evidently to another prism, whose base is the parabolic segment ABCMA, and its height or thickness equal to l, the width of the opening. Now, from the well-known property of the parabola, this segment is two-thirds of the rectangle ABCK, whose area is AB × BC, which has been shown to be equal to $H \times \sqrt{2gH}$; and thus the discharge for the rectangular opening, whose height is H and width l:

$$\frac{2}{3} \times l \times H \times \sqrt{2gH} = \frac{2}{3}\, l \times \sqrt{2g} \times H.\sqrt{H}.$$

47. In the next place, let us seek to determine the discharge through a rectangular orifice opened in the same face, but only from B to D, and having, as before, the constant width, l. Call the charge AD on the upper edge of the orifice h, the discharge of the opening from A to D will also be—

$$\frac{2}{3}\, l \times h \times \sqrt{2gh} = \frac{2}{3}\, l \sqrt{2g} \times h \sqrt{h}.$$

Now it is evident that the discharge from the rectangular orifice, whose height is BD, will be the difference of the discharges by the two openings, AB and AD, into l, and will, therefore, be expressed by—

$$\frac{2}{3} l \sqrt{2g} \left(H \sqrt{H} - h \sqrt{h}.\right)$$

48. *Mean Velocity.*—Let us now return to the determination of the mean velocity, and first, that of the entire rectangular opening. Let G be the point from which would issue the thread with this mean velocity. If we make AG $= z$, it will be equal to $\sqrt{2gz}$, and being multiplied into the area of the opening, $l \times H$, must give the total discharge, which we have already found to be expressed by—

$$\frac{2}{3} . l . H\sqrt{H} \sqrt{2g},$$

we shall therefore have—

$$l \times H\sqrt{2g} \times \sqrt{z} = \frac{2}{3} l \times H \times \sqrt{2g} \times \sqrt{H},$$

and dividing each side by the common factors, $l \times H \times \sqrt{(2g)}$, we have, squaring both sides,

$$z = \frac{4}{9} H;$$

and, consequently,

$$V = \sqrt{2g \frac{4}{9} H} = \frac{2}{3} \sqrt{2gH}.$$

Thus it appears that the mean velocity is equal to two-thirds of the velocity of the lowermost filet; and so GH, which represents the former, is $\frac{2}{3}$rds of BC, which, in like manner, represents the latter. For the case of the rectangular orifice, whose depth is BD or $H - h$, we shall, in like manner, making z' the height due to the mean velocity, have the area expressed by $(H - h) \times l$, and the discharge by—

$$(H - h) \times l \times \sqrt{2gz'} = \frac{2}{3} l \times \sqrt{2g} \left(H \sqrt{H} - h \sqrt{h}\right),$$

Dividing both sides by $l \times \sqrt{2g}$, we have—

$$(H - h) \times \sqrt{z'} = \frac{2}{3} \left(H\sqrt{H} - h \sqrt{h}\right),$$

and dividing by $H - h$, and squaring—

$$z' = \frac{4}{9}\left(\frac{H\sqrt{H} - h\sqrt{h}}{H-h}\right)^2.$$

Example.—A reservoir has on one side a rectangular orifice, 2 feet wide and 1.4 feet deep; the surface of the water being maintained at a constant height above the sill of 2 ft. $= H$; required the height z' due to mean velocity of the flow. We have then $H = 2$ feet and $h = 2 - 1.4 = 0.6$ ft., and

$$(a) \quad . \quad . \quad . \quad z' = \frac{4}{9}\left(\frac{2\sqrt{2} - 0.6\ \sqrt{0.6}}{2-0.6}\right)^2 = 1.2664 \text{ ft.};$$

and as $(H + h) \div 2 = 1.3$ ft., the difference .0336 feet shows the error of the simpler formula when $H = 2$ feet; had it been 6 feet, all else being the same, then from (*a*) $z' = 5.29$ feet, and as $H + h \div 2 = 5.3$ feet, the error is now only 0.01 feet. The mean velocities are in the first case $\sqrt{2g \times 1.2664} = 9.0308$ ft. and $\sqrt{2g \times 1.3} = 9.15$ feet, in the latter they are 18.4556 feet and 18.475 feet.

49. *The charge must be measured from the level of still water.*—It may be proper here to observe, that during the flow of water through an orifice the surface of the water in the reservoir takes a curved form for a certain distance, and bends towards the side in which the orifice is opened; so that the vertical height of the surface above any point in the orifice, estimated beyond the place at which this curvature commences, is greater than that at the side and immediately over the orifice. The former of these heights or charges is that which must be introduced into the formula for the discharge, as will be mentioned in § 55. By overlooking this, as has been frequently done, an error is introduced into the calculation of the discharge, which is thus deficient; and in some cases, though very rarely, it may amount to a tenth part. The error diminishes as the charge increases, and, according to Poncelet, becomes insensible when the charges exceed 0.5 feet to 0.66 feet. This, however, refers to the small experimental orifices used by him. D'Aubuisson states that he measured the depression of the surface at the lock-gates of the canal of Languedoc, and found it, when both sluices were open, from 0.1312 feet to 0.1640 feet.

50. *Coefficient for the reduction of theoretic to actual discharges.*—The discharges of which we have spoken (§ 46) are the theoretic discharges. In order to reduce them to the actual

discharges, it is necessary to multiply them by coefficients deduced from experiments. MM. Poncelet and Lesbros have determined them as in the Table following:—

Charge on the Centre.	HEIGHT OF THE ORIFICES.					
	Feet. 0.656	Feet. 0.328	Feet. 0.164	Feet. 0.098	Feet. 0.065	Feet. 0.032
Feet. 0.032						0.712
0.065				0.644	0.667	0.700
0.098				0.644	0.663	0.693
0.131			0.624	0.043	0.661	
0.164			0.625	0.043	0.660	
0.196		0.611	0.627	0.642		
0.262		0.612	0.628	0.640		
0.328		0.613	0.630	0.638		
0.393	0.592	0.614	0.631			
0.492	0.597	0 615	0.631			
0.656	0.599	0.616	0.631			
0.984	0.601	0.617				
1.640	0.603	0.617				
3.281	0.605					

51. The numbers given above are the true coefficients of contraction of the fluid vein, or coefficients of reduction of the theoretic to the actual discharge; for theory gives no other general formula for the flow of water through orifices than—

$$\frac{2}{3} l \times (H\sqrt{H} - h\sqrt{h}) \times \sqrt{2g}.$$

That which has been established in § 13, namely, $S \times \sqrt{2gh'}$, in which $h' = \frac{1}{2}(H+h)$ is only true in some particular cases—but which no doubt are of very frequent occurrence—in which h' is three or four times greater than $H-h$ (Fig. 17). In all others it is to a certain extent erroneous; and the coefficients adapted to that formula. and which it has served to determine, are so also: those, namely, in the Table § 21, p. 15, which are above the transverse line in each column. The coefficients that are below these lines, though determined by the aid of the formula, $S\sqrt{2gh'}$, are coincident with those derived from the general formula, and are correct. Moreover, in the equation $Q = m S\sqrt{2gh'}$, the error of the coefficient m is compensated by that of the formula, and the discharges assigned by it are sensibly identical with those of the true general expres-

sion; and as it has the advantage in simplicity, it is generally employed in every case.

52. EXAMPLE.—Required the discharge through a rectangular orifice $0^m.30$ wide and $0^m.15$ high, and having a charge of $0^m.05$ only upon the upper edge, we have $H = 0^m.05 + 0^m.15 = 0^m.20$ and $l = 0^m.30$. The charge upon the centre is therefore $0^m.125$, and the corresponding coefficient, according to the above Table, is nearly 0.603,—the mean between 0.592 and 0.614. Thus the discharge will be—

$$\frac{2}{3} \times 0.603 \times 0.30 \times 4.43 \left(0.20 \sqrt{0.20} - 0.5 \sqrt{0.5}\right) = 0^{mmm}.0418.$$

The usual formula $Q = m\,S \sqrt{2gh'}$, with $m = .592$, derived from the Table § 21, gives—

$$0.592 \times 0.30 \times 0.15 \times 4\,43 \sqrt{0.125} = 0^{mmm}.0417.$$

53. *Flow of water over waste-boards, weirs, &c.*—If a rectangular opening, with horizontal base, be formed at the top of one of the sides of a basin, the water—supposed to be maintained at a constant level—will flow out in the form of a sheet over this base or sill: such are the waste-boards of canals and reservoirs, and also weirs in rivers, which extend across their course, so that the water must, when it meets with them, rise, and flow over the crest or summit.

The surface of the water (Fig. 18), before it arrives at a waste board, and from a point C, a short distance above it, is inclined in a curved line, CD, so that its height, immediately above the sill, is not equal to AB, but only BD.

54. If we followed the usual theory, we should at once grant that the particles which followed the curve CD had the same velocity when arrived at D as if they had fallen freely from the height AD, and that all the particles in the vertical line under it would in like manner flow out with velocities due to their heights, below the point A: we should thus find, that in respect to the issuing velocity of the fluid threads, and also in respect to their number,—which depends on the height BD,—and consequently in respect, also, of the discharge, that the case would be identical with that of a rectangular orifice closed on the upper side as far as D, and in which the water level extended without any curvature up to A; and, therefore, if we should put Q for the volume of water discharged in one second, l for the width of the opening, and H and h for the charges,—the one on the lower, the other on the upper edge, —and lastly, m for the coefficient of the reduction of the theo-

retic to the actual discharge, we should have the expression, as in § 47,—

$$Q = \frac{2}{3}\sqrt{2g} \cdot m \cdot l\,(H\sqrt{H} - h\sqrt{h}).$$

55. However natural it would appear to proceed thus, the facts have shown that the discharges are more exactly given by a calculation based on the supposition that the flow took place as if from the whole height AB, the level of the water being supposed continued without curvature up to A. The case then would be identical with that of § 46: for when $h = 0$ we should have—

$$Q = \frac{2}{3}\sqrt{2g} \times m \times l \times H\sqrt{H} = 5.35\ mlH\sqrt{H}.$$

The flow of water, then, over weirs or waste-boards, is only a particular case of the flow by orifices in general; that, namely, in which the charge over the upper edge is equal to zero. MM. Bidone and Poncelet had already shown that this was so, and that the coefficients m, which suit ordinary orifices, are adapted to weirs also, when the flow is made under analogous circumstances.

56. When establishing the formulæ given above, it was assumed that the water was perfectly free from any current above the waste-board edge, or rather above that point at which the surface begins to be curved towards the sill; but frequently the water arrives at this point with an initial velocity; in this case we proceed as we have already done in § 32, for the case of orifices properly so called: that is, we add to the head due to the velocity in the case when the water is in repose (and which is now only $\frac{4}{9}H$, *vide* § 48), the head which would generate the velocity with which it arrives. Let u represent this velocity, and h the charge due to it; then—

$$\left(\text{since } h = \frac{u^2}{2g} = \frac{1}{64.4} \times u^2\right)$$

we have $h = 0.01553\ u^2$, (as $\frac{1}{64.4} = 0.01553$),

representing the additional head in feet; and we shall have, for the actual velocity of the issuing water,—

$$\sqrt{2g\left(\frac{4}{9}H + h\right)} = \sqrt{2g\left(\frac{4}{9}H + 0.01553\ u^2\right)},$$

which may be reduced to—

$$5.35 \sqrt{H + 0.03494\, u^2};$$

and consequently,—

$$Q = 5.35 \times m \times l \times H \times \sqrt{H + 0.03494\, u^2}.$$

The quantity u represents the *mean* velocity of the section of the water approaching the edge. The exact determination of it is, perhaps, impossible; but as its value will be very little different from that of the *surface* (which can be readily determined by methods to be subsequently pointed out), we may, therefore, grant the truth of the equation, and modify the value of the coefficient that is to be determined experimentally. If, then, we put m' for this new coefficient, and w for the velocity of the surface of the water, we shall have—

$$Q = 5.35\, m' \times l \times H \sqrt{H + 0.03494\, w^2}.$$

57. We shall now put these formulæ to the test of experiment. The expression for the discharge contains two quantities, which vary in the experiment,—the *width* of the overfall, and a function of the velocity; that is, of the *charge*. Now, in order that the formulæ be well founded, it is necessary that the discharge be exactly proportional to each of these; then, and then only, will the coefficients be constant. The degree in which they are so in the experiments will thus be a test of their being well founded in truth. The numerous experiments of M. Castel, engineer to the water-works of Toulouse, are quoted by D'Aubuisson, and a description of his apparatus is also given. At one extremity of a wooden canal or trough (Fig. 19), 19.68 feet long, and 2.427 feet wide, and 1.804 feet deep, he received the supply of water, and at the other he had the power of placing different thin plates of copper, in which the overfalls were cut out; their width was increased gradually from 0.0328 feet up to 2.427 feet; the sill being maintained always 0.557 feet above the bottom of the canal. The water discharged was received at pleasure, and for any required length of time, in a cistern lined with zinc, the capacity of which was 113 cubic feet: this was the gauge basin; it was marked with the greatest care by a vertical scale. The time that the water might take to reach any height was taken by a chronometer marking quarter seconds.

58. The charges or heights of the water above the sill of the overfall were successively increased from 0.0984 feet up to 0.328 feet, and even to 0.787 feet for the narrower overfalls. The most important, and at the same time the most difficult point in the experiments, was to measure the charges exactly. In

order to accomplish this, M. Castel placed over the centre of the canal, and parallel to its length (Fig. 20), a bar truly horizontal, which carried at every $0^m.05 = 0.164$ feet, ten vertical brass rods, divided into millemetres, pointed, and having a power of sliding up or down: on the edge of the guides was a vernier, which subdivided the rods into tenths of a millemetre. When an experiment was made, the requisite quantity of water was admitted into the canal; and the regime or regulation for due supply of this being carefully attended to, he let down the rods, and put the points as accurately as possible in contact with the surface of the current. Subtracting then their several readings from the vertical distance between the horizontal bar and the sill, he obtained the ordinates of the curve described by the particles of the fluid as they advanced to the centre of the overfall. These ordinates increased in proportion to their distance from it; and thus the greatest ordinate, or the true charge, H, was obtained; the smallest—that, namely, which was immediately above the sill—was $H - h$, or the thickness of the sheet of water at the moment of its passage over the edge of the waste board.

59. After having to some extent exhausted all the observations which could be made with the canal of $0^m.74 = 2.427$ feet in width, M. Castel entered upon those in which he used a width of only $0^m.361 = 1.180$ feet (Fig. 21), narrowing up the former by two partitions in plank, but whose length was only 7.347 feet, the total length of trough or channel being 19.68 feet. At the entrance A of this small canal, which was placed in the centre of the larger, there was formed with the large discharges a minute fall, which would have introduced some slight modifications into the results obtained if the partitions were extended up to the extremity of the larger canal. Upon both the one and the other M. Castel has made a long series of experiments. Each observation has been repeated once or twice: altogether he has made 494. In every case, the values of Q, of l, and H, were determined directly by the experiment; and from them it was easy to deduce the value of the coefficient m from the formula—

$$Q = 5.35\, mlH\sqrt{H} \text{ for } m = \frac{Q}{5.35 \times l \times H \times \sqrt{H}}.$$

60. The mean values obtained from each charge and each width of overfall are arranged in the two following Tables. No observations have been made in those cases in which there are blanks.

Charge upon the Sill.	CANAL—2.427 Feet wide. COEFFICIENTS, the length of Overfall being											
	Feet. 2.42	Feet. 2.23	Feet. 1.96	Feet. 1.64	Feet. 1.31	Feet. 0.98	Feet. 0.65	Feet. 0.32	Feet. 0.16	Feet. 0.09	Feet. 0.06	Feet. 0.03
Feet. 0.78								0.595	0.615		0.639	
0.72								0.594	0.614		0.639	
0.65							0.596	0.594	0.614	0.629	0.640	0.670
0.59							0.595	0.594	0.613	0.628	0.641	0.672
0.52							0.595	0.592	0.613	0.628	0.642	0.674
0.45						0.603	0.593	0.592	0.612	0.628	0.643	0.675
0.39					0.621	0.604	0.592	0.591	0.612	0.628	0.645	0.678
0.32		0.657	0.644	0.631	0.621	0.604	0.593	0.591	0.612	0.627	0.648	0.687
*0.26	0.662	0.656	0.644	0.632	0.620	0.606	0.595	0.592	0.612	0.627	0.652	0.698
0.19	0.662	0.656	0.645	0.632	0.622	0.610	0.604	0.595	0.612	0.628	0.658	0.713
0.16	0.662	0.656	0.644	0.633	0.626	0.616	0.611	0.597	0.613	0.629	0.663	
0.13	0.662	0.656	0.645	0.636	0.632	0.623	0.619	0.604	0.614		0.669	
0.09	0.663	0.660	0.651	0.642	0.646	0.631	0.624	0.618				

Charge upon the Sill.	CANAL—1.180 Feet wide. COEFFICIENTS, the length of Overfall being									
	Feet. 1.180	Feet. 0.984	Feet. 0.656	Feet. 0.328	Feet. 0.301	Feet. 0.259	Feet 0.164	Feet. 0.098	Feet. 0.065	Feet. 0.032
Feet. 0.787				0.619			0.624	0.629	0.647	0.666
0.721				0.615	0.613	0.617	0.620	0.627	0.646	
0.656				0.611	0.608	0.614	0.618	0.626	0.645	0.667
0.590			0.633	0.608	0.606	0.610	0.616	0.626	0.644	
0.524			0.628	0.605	0.603	0.608	0.615	0.625	0.644	0.668
0.459		0.678	0.624	0.603	0.601	0.605	0.614	0.624	0.644	
0.393	0.700	0.666	0.620	0.600	0.599	0.603	0.614	0.623	0.646	0.674
0.328	0.684	0.656	0.617	0.598	0.598	0.600	0.614	0.624	0.648	
*0.262	0.672	0.652	0.616	0.599	0.597	0.599	0.613	0.624	0.654	
0.196	0.669	0.652	0.617	0.600	0.597	0.600	0.613	0.626		
0.164	0.667	0.653	0.620	0.605	0.604		0.614			
0.131	0.668	0.653	0.624	0.613	0.611		0.613			
0.098	0.670	0.665	0.632	0.628	0.625					

61. Let us now examine that very simple and commonly applied formula, $Q = 5.35\, l \times H \sqrt{H}$; and first, as to the proportionality of the discharges Q to the function of the charge, $H \sqrt{H}$. For this purpose let us take the twenty-two series of discharges obtained, each with the same width, but with different charges; let us reduce the discharges of each series to that which they would have been if one of them—that, for instance, obtained under a charge of 0.262 feet—had been taken as unity: let us, in like manner, reduce the series

of values of $H\sqrt{H}$, and place side by side these several series—as is done in this Table under the head of "discharges" —in three columns; the two first having been given by the canal, 2.427 feet in width, with overfalls 1.968 feet, and 0.328 feet in length; the third was from the canal of half the above width, 1.180 feet, with an overfall 0.164 feet long. It results, then, from a comparison of the twenty-two series of discharges with one another, and with the series derived from the function $H\sqrt{H}$—

Charges. H.	Series of Discharges.			Series of	
	1.	2.	3.	$H\sqrt{H}$	$H\sqrt{H} - h\sqrt{h}$.
Feet.					
0.656		3.96	3.98	3.95	4.01
0.590		3.38	3.39	3.38	3.42
0.524		2.83	2.84	2.83	2.87
0.459		2.31	2.32	2.31	2.34
0.393		1.83	1.84	1.84	1.86
0.328	1.40	1.39	1.40	1.40	1.41
*0.262	1.00	1.00	1.00	1.00	1.00
0.196	0.650	0.652	0.650	0.650	0.643
0.164	0.494	0.498	0.495	0.494	0.486
0.131	0.354	0.381	0.354	0.354	0.345

First. That above the charge of 0.196 feet, and even of 0.164 feet,—and excepting some of the higher charges,—the differences between the numbers on the same horizontal line are very trifling, not exceeding the hundredth part; and thus, confining ourselves to the degree of exactness needed in practice, they may be taken as equal, and the ratio of the discharges is identical with that which obtains between the corresponding values of $H\sqrt{H}$.

Secondly. That for charges of 0.164 feet and below it, the discharges decrease in a less ratio than $H\sqrt{H}$, and by so much the less as the charge is less, but only with the mean lengths; for when they are either very small, or, on the other hand, approach that of the width of the canal itself, they are again found to be equal.

Thirdly. In some experiments with high charges,—especially with the wider overfalls,—we perceive also the discharges increase in a less ratio than $H\sqrt{H}$. This fact, which had been scarcely perceptible in a canal of 2.427 feet, becomes very prominent in that of 1.180 feet, in which the water ar-

rives at the overfall with a very considerable velocity. Now, in these cases—and they occur whenever the transverse section of the fluid ($l \times H$) of the overfall exceeds the fifth part of the transverse section of the course of the canal—the discharges should not be expected to increase as $H\sqrt{H}$, but as $H\sqrt{H + 0.03494\,w^2}$, and we cannot any longer make use of the ordinary formula, but of that which has been given in § 56. In fine, when we limit ourselves to the case of overfalls properly so called,—that is, those in which the water has but a very small velocity of arrival,—the value of Q will be found very closely proportional to $H\sqrt{H}$, and in this view the formula is well established.

62. *Ratio of the discharges to the width of overfalls.*—This coincidence of the formula with the experiments will not be so close when we vary the width of the overfall. The discharges, considered with reference to the variable l, do not follow the exact ratio of the widths, however natural it would appear to suppose beforehand that it would be so. Starting from an overfall the full width of the channel, they diminish with its decreasing width, but considerably faster than it up to a certain point, beyond which, on the other hand, they diminish less rapidly than the width. The columns of the Table appended serve to fix the ideas upon this point.

Canal.			
2.427 Feet.		1.180 Feet.	
Width.	Discharge.	Width.	Discharge.
1000	1000	1000	1000
919	911		
811	788	831	807
676	645		
540	507	554	507
405	371		
270	243	277	246
135	121	138	125
68	62		
40	40		
27	27		
13	14		

For the canal of 2.427 feet in width, we have twelve widths, which are to one another in the ratio of the numbers in the first column: in the second we have the order in which the corresponding discharges decrease, and which

were obtained under charges varying from 0.196 feet to 0.328 feet. In the case of the canal 1.180 feet wide, in which ten widths were experimented upon, those only are stated which were nearly the counterparts of those in the other. The series of these ratios point out the fact, that in the two canals the discharges follow the same law in reference to the widths of the overfalls; but it is to the ~~relative~~ widths in their respective channels, and not to the absolute widths.

63. *As to the coefficients.*—Since the discharges are for the same widths sensibly proportional to $H\sqrt{H}$,—when we omit extreme cases,—the coefficients ought to be very nearly constant, and we find they are so as in the Table § 60. In strictness, when we take the coefficients of some one vertical column of the Table we see them—commencing with the higher charges—decrease by very small degrees, in most of the experiments, down to a certain charge, beyond which they increase rapidly; thus we shall have at this particular charge, which is generally about 0.328 feet, a minimum. And since—the charges remaining constant—the discharges decrease at first more rapidly than the widths of the overfalls, and afterwards less so, it follows that under the same constant charge—the widths, commencing at a first width equal to that of the channel itself, being diminished—the coefficients decrease down to a certain point, beyond which they augment. Thus, here also there is a minimum; and it occurs when the width of the overfall is about the fourth part of that of the supplying channel; and so in both the horizontal and vertical lines of the Tables of coefficients there is a minimum; we have, therefore, a common minimum. In its immediate neighbourhood, and for a certain distance, the variations are small; the coefficients in that part differ very little from one another, and may be regarded as constant. But in the other parts of the Table the differences rise to be considerable; they exceed an eighth, or 12 per cent.; so that the discharge by overfalls cannot be given exactly with a constant numerical coefficient in an expression of the form $H\sqrt{H}$. In practice, then, we would require the aid of very extended Tables of coefficients, the preparation of which would demand many hundred experiments. However, the study of the direction in which the coefficients tend gives us the means of abridging this vast labour, and of determining a few simple rules suitable to the different cases which commonly present themselves.

64. *Coefficients and formulæ to be employed.*—We have seen (§ 61) that the expression $l\,H\sqrt{H}$ should not be employed—

on the one hand, when the charges were below $0^m.06 = 0.196$ ft.; and, on the other, when the transverse area—that is, the depth multiplied by the width of the overfall—exceeds the fifth part of the area of the section of the water in the supplying channel, the initial velocity becoming then considerable. Between these limits, the expression above given can be employed with a coefficient,—variable, it is true, but only varying with the width of the overfall.

Reckoning from a width equal to that of the canal itself, the coefficients diminish with the width of the overfall until it has reached the fourth part of that of the channel, and then they increase, although the widths are still decreasing; and what is very remarkable is, that the diminution of the coefficients follows the relative widths of the overfall to the canal, whilst the augmentation, which occurs afterwards, depends only on the absolute width. We have, consequently, four cases to distinguish relative to the coefficient to be used:—

First. In the neighbourhood of that common minimum we have mentioned, the variations of the coefficients are very trifling. According to the experiments made by Castel, from a width of overfall nearly equal to the third of that of the canal supposed to exceed 0.984 feet, down to an absolute width of 0.1640 feet, the coefficients do not vary more than from 0.59 to 0.61. Taking the mean, and remarking that—

$$5.35 \left(= \sqrt{2g} \times \frac{2}{3}\right) \times 0.60 = 3.21,$$

we shall have between the limits indicated above—

$$Q = 3.21\, lH \sqrt{H}.$$

This formula furnishes the best mode of gauging small streams of water, as in the Examples appended.

Secondly. When the width of the overfall is at its maximum,—that is, equal to that of the canal, and is then precisely similar to a weir properly so called,—the coefficients present a remarkable steadiness under the different heads. M. Castel, in his experiments upon the canal of 2.427 feet, with an overfall or barrier of a height equal to 0.557 feet, had no difference in the coefficients obtained with charges which themselves varied from 0.098 feet to 0.262 feet, § 60. With an overfall of 0.738 feet high, the coefficients have only ranged from 0.664 to 0.666 for charges of 0.101 feet up to 0.242 feet: at the mean he had 0.665. And since—

$$5.35 \times 0.665 = 3.55775,$$

we have, putting L for the width of the canal or length of the barrier—

$$Q = 3.558\, LH\sqrt{H}.$$

This formula will be used with advantage in certain cases, even in large water-courses, and with charges of $0^m.04$ and $0^m.03 = 0.131$ feet and 0.098 feet; but, to insure certainty in its use, it is necessary that the charge be less than the third of the height of the barrier.

Thirdly. For widths of overfall comprised between that of the channel itself and the fourth part of the same, the coefficient of the expression $5.35\, lH\sqrt{H}$ will vary with the relative width,—that is to say, with the ratio of the width of the overfall to that of the canal of supply, and is given by the columns of this Table:—

Relative Widths.	Coefficient for Canal of	
	2.427 Feet.	1.180 Feet.
1.00	0.662	0.667
0.90	0.656	0.659
0.80	0.644	0.648
0.70	0.635	0.635
0.60	0.626	0.623
0.50	0.617	0.613
0.40	0.607	0.609
0.30	0.598	0.600
0.25	0.595	0.598

They have been formed by taking proportionals to the coefficients deduced from direct experiments, as given in § 60, a method which cannot here lead to any error. The coefficients determined by each canal have been given separately, in order to show that coefficients sensibly the same correspond to the same relative width, although the actual value of the widths was in one canal nearly double of that of the other, affording a proof that above 0.25, or the fourth of the width of the channel, the coefficients depend on the relative, and not on the absolute width of the overfall.

Fourthly. It is quite otherwise when this width falls below that of the fourth of the supplying canal: then, and when at the same time it is absolutely less than $0^m.08$ or $0^m.06 = 0.262$ feet or 0.196 feet, that of the canal has no further effect, and every particular width has its own coefficient. Thus in the canal of $0^m.36 = 1.180$ feet, as well as in that of $0^m.74 = 2.427$

feet, the widths of 0.164 feet, 0.098 feet, 0.065 feet, and 0.0328 feet, have in both given the respective coefficients 0.61, 0.63, 0.65, and 0.67.

65. *Observations on formula of* § 54.—Having thus given in detail all that has reference to the simplest of the formulæ of discharge for overfalls, let us consider the two others, and first, that of—

$$Q = 5.35\ ml\ (H\sqrt{H} - h\ \sqrt{h}),$$

in which h represents the quantity AD (Fig. 18), the surface of the fluid having become curved before its arrival at the overfall. A slight inspection of the last column of the Table given in § 61 shows, that although the series of values of $H\sqrt{H} - h\ \sqrt{h}$ does not differ much from the series of values of the corresponding discharges, yet that it follows them less closely than the series of values of $H\ \sqrt{H}$. Thus in this important point, the second formula is not so well established as the first. It is, moreover, of much more difficult application, containing an additional term, and one whose exact determination is a matter of very great difficulty.

66. *Observations on the formula in* § 56.—This is, however, not so much the case in that formula, which involves a term which is a function of the velocity with which the water flowing in the canal arrives at the overfall.

It is evident that in the case of a high velocity, in which the flow takes place both from the charge H and from an initial velocity, w, taken at the surface, it is necessary to add to the charge a term depending on this velocity, which leads to the equation § 56.

$$Q = 5.35\ m'lH\sqrt{H + 0.03494\ w^2}.$$

The experiments of M. Castel give the values of m' the coefficient. In these experiments the velocity w of the surface of the water in the canal has not been actually measured, but it can be determined from the mean velocity, which is equal to the discharge Q, divided by the section of the running water in the channel of supply, which in this case is $L \times (H + a)$, representing by L the width of the channel of supply, and by a the height of the sill of the overfall above the bottom of the channel; and as

$$v \times \{L\,(H + a)\} = Q\ \therefore\ v = \frac{Q}{L\,(H + a)}.$$

It will hereafter be shown that the velocity of water at the surface is, on an average, a fourth part higher than the mean velocity; so that we have—

$$w = 1.25 \times \frac{Q}{L(H + a)}.$$

Even with this value of w—which is the highest we may assume—the coefficient m' differs only from the coefficient m in the common formula when the velocity in the channel is great enough to occasion the term $0.0349\ w^2$—which is that which makes the difference between the two formulæ—to have a value comparable to that of H. As it will be in most cases very small, and as it is under the radical sign, it will only influence the value of m' by half its amount relatively to H; for example, if it is the 2, 4, or 6 hundredths of H, the coefficients, *ceteris paribus*, will only differ by the 1, 2, or 3 hundredths. In these three cases the section of the sheet of water at the overfall, or $l \times H$, is found to be respectively equal to 0.1724, 0.244, and 0.3, of the section of the canal of supply, or of $L\ (H + a)$, whence may be deduced the conclusion of which we have already made use (§§ 61, 64), that when the former of these two sections is less than the $\frac{1}{5}$th part of the latter, the coefficients m and m' will be the same within a hundredth part nearly. Such has been the case with the overfalls used by M. Castel, whilst their width has been less than the half of that of the channel. When it was greater the term $0.0349 w^2$ has had more effect, and the differences became larger. But the use of this term is far from having brought to equality the coefficients m and m' for various widths of overfall: it has not reduced even by half the differences which occur in the values of m; and neither the expression—

$$Q = 5.35\ m'lH \sqrt{H + 0.0349\ w^2},$$

nor that of

$$Q = 5.35\ mlH \sqrt{H},$$

can be employed with a constant coefficient except in the case of a width of overfall equal to that of the canal of supply.

In order to obtain the coefficient in this case, M. Castel has dammed up the canal of 2.427 feet by means of barriers of copper, whose height has been decreased successively from $0^m.225$ to $0^m.032 = 0.738$ feet and 0.104 feet; and he has thus obtained the coefficients in this Table:—

Height of the Dam.	Coefficients m', the Charge being			
	0.26 Feet.	0.19 Feet.	0.16 Feet.	0.13 Feet.
Feet.				
0.738	0.651	0.655	0.657	0.660
0.557	0.640	0.647	0.650	0.654
0.426	0.650	0.649	0.652	0.656
0.305	0.635	0.642	0.646	0.650
0.246	0.647	0.652	0.655	0.660
0.134	0.667	0.664	0.665	0.668
0.104	0.676	0.676	0.676	0.680

Those of the first five barriers are nearly the same; and although they do not present the same regularity which we had in ordinary overfalls, we may assume 0.650 as the mean. As to the last two barriers of $0^m.041$ and $0^m.032 = 0.134$ feet and 0.104 feet, they are in a distinct class. They were very low, and the charges very much exceeded their height; so that the case was nearly as much one of water flowing in an ordinary channel, as of it when passing over a waste-board. It may be remarked, moreover, that the near equality between the coefficients for one and the same barrier speaks strongly in confirmation of the formula which has determined them. The experiments upon the canal of 1.180 feet, with its barrier of 0.557 feet, have indicated coefficients whose mean was 0.654. Taking, then, the mean between this and 0.650, that is, 0.652, and observing that $5.35 \times 0.652 = 3.488$, we shall have finally—

$$Q = 3.488\ LH\sqrt{H + 0.0349w^2}.$$

The velocity w is to be determined by direct observation.

67. *Overfalls, with channels attached.*—We frequently find channels are adapted to overfalls: they may be considered as the prolongation of their horizontal and vertical edges. The water discharged is now confined, and suffers a resistance from the friction of the bottom and sides, which retards the motion, and this retardation, re-acting on the water which arrives at the overfall, diminishes the discharge. The following experiments by MM. Poncelet and Lesbros exhibit the effects of this resistance. The additional channel was always $3^m = 9.84$ feet long, and of the same width as the overfall, $0^m.20 = 0.656$ feet, and adjusted so as to be horizontal:—

Charge.	Coefficient.		Loss per Cent.
	Without added Channel.	With added Channel.	
Feet.			
0.675	0.582	0.479	18
0.475	0.590	0.471	20
0.337	0.591	0.457	23
0.196	0.599	0.425	29
0.147	0.609	0.407	33
0.091	0.622	0.340	45

The amount of diminution in the discharge from *overfalls* with the channel attached has therefore been so much the less, as the charge has been higher. From this we may infer, that with charges of 3 or 4 feet and greater, such as are often in operation at the head of large feeders and water-courses, the diminution of discharge due to the resistance of the sides of the channel is inconsiderable. With *orifices* the same experimenters arrived at results analogous to those of overfalls. They applied the additional channel, 9.84 feet long by 0.656 feet wide, mentioned above, to the exterior of the orifices from which had been derived the Table § 21, p. 15, where it is mentioned that the orifices were all 0.656 feet wide; and from numerous experiments it was deduced, that when the charges, measured from the centre of the orifice, were from 2 to $2\frac{1}{2}$ times greater than the height of the orifice itself, the channel attached had no decided influence upon the discharge: it was the same in amount whether this was or was not present; but with very small charges it diminished the charge even a fourth or more. Further investigations as to the effect of inclining the channel were undertaken. When the slope was 1 in 100, or 34′, the coefficients were found the same as when the channel was horizonal, but at 1 in 10, or 5° 44′, they were increased 3 or 4 per cent. Castel also experimented on overfalls, and on his canal of supply, 2.247 feet wide, he placed an overfall 0.656 feet wide, with a channel attached 0.669 feet long, inclined 1 in 13.3, or 4° 18′. The following Table gives—

Charge.	Coefficient.
Feet.	
0.364	0.526
0.311	0.527
0.249	0.527
0.196	0.528
0.164	0.530

the coefficients obtained with the formula, $Q = 5.35\ mlH\sqrt{H}$. They vary very slightly, although the charges have more than doubled. The mean is 0.527, and would probably have been 0.53 if the inclination had been 1 in 12, which is common in practice. With the simple overfall the coefficient was 0.60; so that the additional channel diminished the discharge about 12 per cent.

68. A particular case yet remains to be considered—namely, that of the *demi-deversoirs* or *deversoirs incomplets*, as Dubuat has called them, or drowned weirs in English writers, so called when the tail water has risen above the level of the sill. Dubuat has divided the height of the water above the sill into two parts (Fig. 22), Ab and bC. In the former, the flow takes place as in an ordinary overfall, in which Ab ($= H$) is the charge; so that the volume of water discharged (§ 66) is expressed by $Q = 3.488\ lH\sqrt{H + 0.0349\ w^2}$. In the second portion it may be assumed that the discharge is the same as in a rectangular orifice, of which bC is the height, and the charge equal to the difference of level Ab between the upper and lower surface of the water. bC is the height of this latter surface above the sill of the overfall, and if we call the height bD of the surface above the bottom of the canal, a, and the height CD of the sill above the same point, b, it will be equal to $a - b$. To the charge Ab or H is to be added, as in the case of closed orifices (§ 32), the height due to the velocity u of the water in the canal, and the velocity of issue will be—

$$\sqrt{2g\,(H + 0.01553\ u^2)} = \sqrt{2g\,(H + 0.01\ w^2)},$$

since $w = 1.25\ u$ (§ 66). Thus we shall have for the discharge of this part (§ 29)—

$$Q = 4.96\,l.(a - b)\sqrt{H + 0.01\ w^2},$$

adding the two partial discharges, and putting Q for the total discharge—

$$Q = 3.488\, lH\sqrt{H + 0.0349\, w^2} + 4.96\, l\,(a - b)\sqrt{H + 0.01\, w^2}.$$

69. *Arrangements preliminary to gauging.*—Weirs constructed completely across the bed of a river may sometimes give the means of measuring the discharge; but it is in such cases necessary that the crown of the weir should have a well-defined edge, so that the water which flows over it may fall freely, and without meeting any check from the reaction of the body of water already passed over. (*Vide* § 67.) It is but seldom that they are so constructed; however, we may, without great expense, adapt a weir with the usual rounded crest to purposes of gauging by raising on the crest some temporary structure that shall have the necessary well-defined edge, and of a sufficient height, so that the discharge may not suffer from any such reaction. We must so regulate the length of this apparatus that the depth of the water, H, flowing over may be greater absolutely than 0.1968 feet, § 61, but less relatively than the fourth part of the depth of the river as it approaches the weir. In such cases the discharge will be given by the formula $Q = 3.558\, LH\sqrt{H}$ (§ 64), L being the length of the temporary crest or edge. In case H should exceed the fourth part of the depth of the current of the stream, we must use the formula in § 66, $Q = 3.488\, LH\sqrt{H + 0.0349 w^2}$, w being the velocity taken at the surface.

70. If the mode of gauging by weirs be but seldom applicable to larger rivers, it is, on the other hand, the most suitable for small streams and water-courses. We have two cases to consider: first, that in which the quantity of water discharged is at or under, about 40 cubic feet per second. We must seek a spot where an overfall can readily be established, having a length, L, greater than 0.3 feet, but less than the third part of the width of the bed of the stream, and so disposed that we may have a charge, H, greater than 0.1968 feet; all being, moreover, subject to the condition that lH be not greater than the fifth part of the section of the current immediately above the overfall; then, without fearing any error greater than the hundredth part, we may apply the formula, $Q = 3.21\, lH\sqrt{H}$ (§ 64).

If, secondly, the quantity discharged should exceed 40 cubic feet per second, we must pond up the water by a dam extending from bank to bank, and at each extremity place vertical side-boards, so that the opening traversed by the water

may be rectangular, the crest being truly horizontal, and using either of the formulæ mentioned above according to the conditions specified. The two examples following will point out the manner of proceeding, and furnish an opportunity of adding some practical details, elucidating what has been laid down in § 63 to § 66.

71. EXAMPLES.—1st. Let us suppose it necessary to gauge the discharge of a small river or water-course: we must search for a part suitable for the construction of an overfall. This will probably be found at a point where the bed has become contracted, and the banks are somewhat steep, and immediately below a wide part of the stream; at such locality the width at the surface of the water is found to be 11.808 feet, and the greatest depth 2.62 feet. After a preliminary examination of the transverse section, and of the surface velocity, measured by means of some light body thrown into the current, we find, approximately, multiplying the assumed section by the velocity, that the stream is discharging nearly 35 cubic feet per second.

Since the width is 11.808 feet, we may give 3.936 feet to that of the overfall for gauging (i. e. 11.808 ÷ 3); the charge H will then be above 0.1968 feet, for the formula $Q = 3.21\, l H \sqrt{H}$ gives—

$$H^{\frac{3}{2}} = \frac{Q}{3.21 \times l} \text{ or } H = \sqrt[3]{\left(\frac{Q}{3.21 \times l}\right)^2}$$

which, for the assumed discharge, gives in this case,

$$H = \sqrt[3]{\left(\frac{35}{3.21 \times 3.936}\right)^2} = \sqrt[3]{(2.7703)^2} = 1.9725 \text{ feet.}$$

From this preliminary inspection we may construct a suitable partition of plank about 0.1 feet thick on the upper edge, and of such figure as nearly to fit the sides and bottom of the water-course. It must be carefully stanched, being sunk into the bottom and sides, and puddled on the up-stream face. From out of its upper edge we must cut an opening 3.936 feet long and 2.132 ft. deep, so that its sill being 0.488 feet above the bottom of the bed (2.62 − 2.132), the water may flow off freely. The section of the presumed discharge (3.936 feet × 1.969 = 7.746 square feet) being not the fifth nor even the seventh part of the transverse section of the river, which exceeds 55 square feet, all the conditions for the application of the formula $Q = 3.21\, l H \sqrt{H}$ are present. Everything being duly prepared for the gauging of the water—such as, all leakage having been stopped, and the current restored to its

ordinary and uniform flow, we proceed to measure H by stretching a cord across the opening, whose ends are fastened to points in the sides, marked at the level of still water (deduction being made for capillarity), and about a foot from the vertical side of the overfall. The depth of the sill below this line at the centre of the opening is carefully measured, and found to be 2.01 ft., and the length also, intended to be 3.936 feet, is found to be 3.92 feet. The discharge is, therefore, $Q = 3.21 \times 3.92 \times 2.01 \sqrt{2.01} = 29.128$ cubic feet.

72. 2nd. A question of law requires that the exact quantity of water flowing down a stream when the surface is level with the top of a certain fixed mark be determined. The gauging must evidently be effected by a dam across the course. About 170 feet above the mark a temporary dam is placed, at a part where, from its regular width and inclination, the river-bed is suitable, having, when the water is at the height above named, a breadth of 64.94 feet, and a mean depth of 4.1 feet; the overfall being a plank well squared, and 0.1312 feet thick, the upper edge being placed truly horizontal, and 0.656 feet above the bench-mark. At each extremity a vertical piece is raised, so that the length of the overfall is 63.6 feet; close to the vertical pieces two others are placed, on which a scale is drawn whose zero is the upper edge of the plank. These arrangements being made, it is only necessary to observe when the surface of the water down stream is level with the fixed mark, and then read the height of the water upon each scale. This last has been found to be 2.34 feet. As this height is nearly the half of that of the temporary dam ($4.1 + 0.656 = 4.756$), we cannot apply with confidence the formula $3.558\, LH \sqrt{H}$, § 64, p. 44, and we must use—

$$3.488\, LH \sqrt{H + 0.0349\, w^2}, \text{ § 66, p. 47.}$$

To obtain the value of w, the velocity of the surface of the current on arriving at the overfall, we must take a distance of, say, 165 feet on each bank, above the point where the surface of the current begins to curve towards the overfall, and, marking these points, we must place in the current, about 60 or 70 feet above them, some floating body of the same specific gravity as water, and mark carefully the time which it may take to flow along the 165 feet: a mean of six observations gave $48\frac{1}{2}$ seconds; whence—

$$w \text{ is } = \frac{165}{48.5} = 3.4 \text{ feet per second, and } 0.0349\, w^2 = 0.40344.$$

Hence—

$$Q = 3.488 \times 63.6 \times 2.34 \sqrt{2.34 + 0.40344} = 859.78 \text{ cubic feet,}$$

the formula $3.558\, LH \sqrt{H}$ would give 810.63 cubic feet. Thus we may certify, that at the given height of surface the river discharges about 850 cubic feet of water per second.

73. Experiments on weirs on a large scale have been undertaken by Mr. T. E. Blackwell. The first set are contained in the following Table, which has been arranged from the detailed experiments, as published in the Civil Engineer and Architects' Journal, vol. xiv., p. 642:—

FIRST SET OF EXPERIMENTS—T. E. BLACKWELL.

Length of Overfalls.	Coefficients under their respective Charges.												Mean Coefficients.
	Feet. .083	Feet. .166	Feet. 0.25	Feet. .333	Feet. .416	Feet. 0.50	Feet. .583	Feet. .666	Feet. 0.75	Feet. .833	Feet. 1.00	Feet. 1.166	
	Overfall being a Thin Plate .0052 Feet.												
Feet. 3	.676	.675	.630	.616	.601	.592							.631
10	.808	.802	.642	.655	.649			.580	.529				.667
	Overfall Plank, 0.166 Feet thick												
3	.466	.508	.562	.549	.588	.592	.616	.606	.600	.613			.570
6	.459	.561	.597	.574	.601	.607	607	.589	.568	.538	.525	.549	.565
10	.435	.585	.568	.601	.609	.576	.571	547	.558		.534	.534	.556
	Same Overfall, with Wingboards, converging at Angle 64°.												
10	.754	.675		.655	.670								.688
	Crest 3 Feet long, sloping 1 in 12.												
3	.466	.532	.538	.454		.531	.526		.498				.507
	Crest 3 Feet, sloping 1 in 18.												
3	.544	.546	.537	.430	.516		.513	.490	.492				.508
10	.466	.495		.514		.543		.507					.505
	Crest 3 Feet, level.												
3	.451	.481	.441	.418	.478	.501	.487	.469	.475				.466
6			.492	.496			.496			.465	.466		.483
10	.381	.478			.517	.513		.468	.486	.454			.471

This Table is the result of a series of 243 experiments, made on overfalls of 3 feet, 6 feet, and 10 feet in width, with heads from 0.0833 feet to 1.166 feet, and with the varying circumstances of having for the overfall bar,—first, a thin plate; secondly, a plank 0.166 feet thick, square on the top; and thirdly, a crest 3 feet in breadth. The thin plate spoken of as forming one of the overfall bars was a piece of iron fender plate, barely 0.0052 feet thick; and the broad crest used was an apron formed of deal boards 3 feet long, roughly planed over, and fastened on to the outer edge of the vertical overfall plank, so as to form an uninterrupted continuation of it, the object being to approximate towards the case of well-constructed wide-crested weirs, such as may be found in actual use in rivers, &c.: the position of this planking was, in some of the experiments, horizontal, and in others sloped at 1 in 18 and 1 in 12.

The second set consisted of a series of 70 experiments on an overfall about 10 feet in length, the width of the bar being in every case 0.166 feet.

74. In both, special circumstances which influenced the coefficients obtained were present; and in employing them in actual practice, a careful judgment must be formed as to their exact degree of applicability. The former were made upon a side pond or reservoir of the Kennet and Avon Canal, the area of which was 106,200 square feet; and as to these the author remarks, first, that the pond was supplied with water, not continuously, as drawn off in the experiments, but three or four times a day, or as often as might be requisite: however, even if during an experiment no water had been admitted, the decrease of the "head" must have been inappreciable, for the largest discharge measured in any experiment was barely 444 cubic feet, which gives a fall of ($444 \div 106200 =$) 0.00418 feet over the whole surface, between the beginning and end of it, and the half of this, or 0.00209 feet, deducted from the head,—measured at the commencement,—for the approximate mean head, could not be perceived. Secondly, at some little distance above the overfall the depth of water was reduced by a submerged course of masonry (Fig. 23), which rose to within 18 or 20 inches of the surface: the width of the approach to the overfall being about 40 feet, we have a transverse area of 60 square feet. Thirdly, the overfall was placed upon the outer line of the dam, in order to obtain the requisite free fall; and the depth of water immediately contiguous to it and on the dam would seem, from the section given, to be a little more than 2 feet; the average depth between the submerged course and the dam

being about 3 feet. The line of the overfall, moreover, was not exactly in the direction of one of the sides of the reservoir; and from the measurements given of the head, taken at still water, and the corresponding depth of the sheet of water flowing off, as given in the following Table, it would appear that some degree of resistance opposed the motion of the water up to the overfall.

75. *Actual depth at point of discharge.*—In this Table the actual heads were measured at the outer edge of the boarded crests and edges of the overfalls. The upper line of figures gives the head, H, at still water, in feet; the other figures the heads at the outer edge of overfalls or crests, also in feet.

	Overfall.	Head at Still Water.									
		Feet. .083	Feet. .166	Feet. .25	Feet. .33	Feet. .416	Feet. .5	Feet. .583	Feet. .666	Feet. .75	Feet. .833
		Resulting Depths at Outer Edge.									
Overf. plank 2 in. thick.	3 ft. long,	.052	...	...	.25 to .224	.291	.364	...	.531	...	...
	6 ft. long,	...	...	.161	.291	.296	.361	...	...	...	...
Crests of boards, 3 ft. wide.	3 ft long level, ...	.036	.073	...	.114	.187	.198	...	...	...	...
	3 ft., slope 1 in 18, .	...	...	...	.114	.145	...	.177	...	...	...
	3 ft., slope 1 in 12, .	.0208	...	...	.125	...	...	.198	...	...	...
	6 ft. long level, .	.041	...	.093 to .104	...	...	.229	...	...	.343	...
	10 ft. long level, ...	.036	.070	...	...	.145	.218	...	.302	.291	.333
	10 ft., slope 1 in 18, .	.026	.046	...	.125	...	.187	...	.291	...	...

The general arrangements at these, the first set of experiments, would seem fairly to represent the case of the discharge of water by an overfall from a large still reservoir.

76. In the second set of experiments—those at Chew Magna—we have an area of reservoir of 5717 square feet kept constantly full by a pipe 2 feet diameter, discharging from a head of 19 feet. The distance between the mouth of this pipe and the overfall was only about 100 feet; the water must, therefore, have retained some of its velocity on approaching the overfall; and indeed, with charges above 0.417 or 0.5 feet, this was perceptible to the eye, but could not, the author states, be accurately determined, from the peculiar form of the reser-

voir. The results, however, show that this influence must have been considerable, and that the effect of water approaching an overfall with an initial velocity is an element which should never be disregarded. The longitudinal section, Fig. 24, as compared with that in the first set, at the Kennet and Avon Canal reservoir, seems also more favourable to the free approach of the water. The overfall had wings at an angle of 45°, well adapted for facilitating the discharge. The overfall bar was a cast-iron plate, 0.166 feet thick, with a square top. The general circumstances attending these, the second set of experiments, make the discharges given by them analogous to the case of a weir in a river, or in a running stream; and in this view they have great value when carefully applied.

SECOND SET OF EXPERIMENTS—T. E. BLACKWELL.

Overfall, Cast-iron Plate, 0.166 Feet thick; Length, 10 Feet.					
Head in Feet.	Coefficients.	Head in Feet.	Coefficients.	Head in Feet.	Coefficients.
.083 to .073	.591	.3437	.743	.5	.749
.083 to .088	.626	.3594	.760	.5156	.748
.182 to .187	.682	.3646	.741	.5156 to .521	.747
.229	.665	.3610	.750	.5781	.772
.2435	.670	.375	.725	.639	.717
.2396	.655	.416	.780	.6666	.802
.2422	.653	.4227	.781	.66 to .734	.737
.2448	.654	.4505	.749	.7448	.750
.25 to .253	.725	.453 to .456	.751	.75	.781
.3333	.745	.4948	.728	Mean.	.723

77. These several sets of experiments are probably, as to length of overfall, and charges above 0.75 feet, the largest that have been yet recorded. To compare them fully with those of D'Aubuisson, it would, perhaps, be necessary to have had a greater number of widths below that of 10 feet than 6 and 3 ft., but with these only, as published, the comparison tends, in some degree, to confirm the coefficients given by D'Aubuisson in §§ 64, 66. Let us, for this comparison, recapitulate what has there been laid down. First, from the Tables, § 60, and the remarks, § 64, it appears that, with an overfall whose length is one-third or less of the channel of supply, we must use 0.60 as the multiplier or coefficient to reduce the expression—

$$\frac{2}{3} l H \sqrt{2 g H},$$

so as to give the true discharge per second (page 43). Hence,

$$Q = 3.21\, l \times H \sqrt{H}.$$

Secondly, when the overfall is of a length equal to that of the width of the channel of supply, we may use 0.665, provided also, that the head of water be less than one-third of the height of the dam above the bottom of the channel of supply, giving—

$$Q = 3.558\ LH\sqrt{H}.$$

Thirdly, we have coefficients which decrease from 0.662 to 0.595, in proportion as the length of the overfall ranges from the full width of the channel to the fourth part of it. Lastly (§ 66, p. 47), we have 0.652 to be employed with overfalls whose length is the full width of the channel of supply; and when also the initial velocity of the water in this channel is represented in the formula by the quantity added to H in the factor—

$$\sqrt{H + 0.0349\,w^2}, \text{ giving } Q = 3.488\ LH\sqrt{H + 0.0349\,w^2}.$$

We have also, in the case of the additional channels or broad crests, the coefficient 0.527 to be used for m in the formula—

$$Q = 5.35\ mlH\sqrt{H}, \text{ giving } Q = 2.8355\ lH\sqrt{H}.$$

78. In Mr. Blackwell's experiments we can only use for this comparison those in which the overfall was constructed of plate-iron 0.0052 feet thick; and secondly, those having the crests 3 feet broad attached below the edge; since D'Aubuisson has only recorded experiments made with a thin plate of copper for the overfall, and those which had a channel attached below the edge. Now, if we consider the 10 feet overfalls of Mr. Blackwell as being analogous to those in D'Aubuisson, in which the length is equal to the full width of the channel of supply,—which the plan (Fig. 25) would seem to justify,—we find that, taking out from Table § 73 the average coefficient, overfall of thin plate-iron and 10 feet long, we have—

According to T. E. Blackwell,	. . . . .	0.667
„	D'Aubuisson (§ 64, second case),	0.665

The average of the 3 feet overfall (less than one-third of the channel of supply), constructed of plate-iron 0.0052 feet thick, according to Blackwell, is 0.631, in the "first case" in § 64 we find 0.600 as being used by D'Aubuisson in analogous circumstances. It may, however, be observed, that the charges in Mr. Blackwell's experiments, which give 0.631, not going higher than 0.5 feet, and the coefficients decreasing up to that head, make it probable that they would have decreased much lower had the experiments been continued, and so reduced the coefficient 0.631 to a value nearer to 0.600.

Again, in the experiments with the added channels or broad crests, we find the average of Mr. Blackwell, when the crest is horizontal, to be 0.473. The average of those in D'Aubuisson, § 67, p. 48, is 0.430, but particular experiments give a closer agreement in the coefficients: for instance, if we take out that derived from the charge 0.337 feet in this last Table the coefficient is 0.457; and in § 73, under the nearly equal charge of 0.333, we have the same identical coefficient 0.457 as the mean of the two given for the 3 feet and 6 feet overfalls. Castel's experiments for overfalls, with channels attached, sloping 1 in 13.3, give at the mean 0.527; the mean of those in § 73, sloping 1 in 12, is 507. In these also, if we take out the particular heads of 0.164 feet in the former, and 0.166 in the latter, we have the respective coefficients 0.530 and 0.532.

79. The overfalls having the bar in the first set of experiments of plank, and in the second set at Chew Magna, of cast-iron, each 0.166 feet thick, and with square-top edges, represent a very common structure for waste weirs, tumbling-bays, &c., on artificial canals and feeders. The average of the first set, with charges from .083 to 1.166, and 10 feet length of overfall, gives 0.556; that of the second set is as high as 0.723. The plan and longitudinal section of the channel of approach is evidently more favourable in the latter case than the former; and the velocity of the approaching water must also have been considerable, from the circumstance mentioned in § 76. The very low coefficient 0.556 is not, however, readily to be accounted for; nor is it easy to assign a reason why, in the first set, the change from a thickness of 0.0052 feet to 0.166 feet in the overfall bar should lower the coefficients to such an extent, every other circumstance being apparently the same. If we look to the coefficients of particular experiments, as, for instance, under charge .083 feet, length 10 feet, we find .808 with overfall .0052 feet thick, and only .435 with that whose edge was the plank 0.166 feet thick; and again, under charge of 0.75 feet, with 10 feet lengths, we have 0.529, for overfall 0.0052 thick, and 0.558 for that 0.166 feet thick,—that overfall having in this case the lesser coefficient, which had in the first instance one nearly double; again, with the 3 feet overfalls, head 0.50, the coefficient is the *same in both*,—namely, 0.592.

80. The following Table, compiled from various sources, exhibits at one view the results of different experimenters:—

Overfall, 0.5 feet long.—Smeaton and Brindley.												
Heads,			.083	.1042	.1146	.1354	.1927	.2604	.4166	.4687	.5417	
Coeffs.			.713	.681	.654	.638	.636	.602	.609	.571	.633	
Overfall, 1.533 feet long.—Du Buat.												
Heads,						.1482		.2666	.3887		.5627	
Coeffs.						.648		.624	.627		.630	
Overfall, 0.656 feet long.—D'Aubuisson and Castel.												
Heads,			.098	.131	.164	.196	.262	.328	.393	.459	.524	.590
Coeffs.			.632	.624	.620	.617	.616	.617	.620	.624	.628	.633
Overfall, 0.6458 feet long.—Poncelet and Lesbros.												
Heads,	.033	.066	.099	.1332	.1998	.2664	.333	.5	.666	.75		
Coeffs.	.636	.625	.618	.611	.601	.595	.592	.590	.585	.577		
First Set.—Overfall, 3 feet and 10 feet long.—Simpson and Blackwell.												
Heads,			.083	.166	.25	.33	.416	.50	.583	.666	.75	
Coeffs.			.742	.738	.636	.635	.625	.592		.580	.529	
Second Set.—Overfall, 10 feet long.—Simpson and Blackwell.												
Heads,			.083	.166	.25	.33	.416	.50	.583	.666	.75	
Coeffs.			.608	.682	.725	.745	.780	.749	.772	.802	.781	

In every case except the last we may perceive that the coefficient decreases as the charge increases. Another exception may be found by referring back to the experiments of the first set,—overfall being a plank 0.166 feet thick,—in which the coefficients increase with the increase of the charges; the lengths being 3 feet, 6 feet, and 10 feet; and with the lengths of 6 feet and 10 feet attaining a maximum value at the charge of .583 feet, from which they slightly decrease.

81. *Method of determining the coefficient from experiments.*—Smeaton's experiments were conducted by making observations upon the time in which a vessel of 20 cubic feet capacity was filled by the water flowing over a notch 0.5 feet long, and with the different charges given in the Table above. (Encyclopædia Britannica, Hydrodynamics, § 212. 7th Ed.)

Thus, with 0.1042 feet charge we should have from the formula—

$$\frac{2}{3} l H \sqrt{2g H} = \frac{2}{3} \times 0 5 \times 0.1042 \times \sqrt{0.1042} \times 8.024 = 0.08995$$

cubic feet per second; but the experiment gives 20 cubic feet in 326 seconds, or ($20 \div 326 =$) 0.06135 cubic feet in one second, and—

$$.08995 : .06135 : : 1 : .682 \left(= 1 \times \frac{.06135}{.08995}\right).$$

So also: Du-Buat had a gauging reservoir to receive the discharged water, whose area was 108 square feet French; the water discharged by a notch (*reversoir*) 17 pouce 3 ligne long, with a head of 1 po. 8 lig., raised the surface of the reservoir above mentioned 5 po. in three minutes: hence the discharge per second was $15552 \times 5 \div 180 = 432$ cubic pouce, and the formula gives—

$$Q = \frac{2}{3} \times 17.25 \times 1.666 \sqrt{1.666} \times \sqrt{2g};$$

and as with this unit $2g$ is equal 724, we have $\sqrt{2g}$ equal to 26.907, the resulting value is 665.3 po. cubic, and 665.3 : 432 : : 1 : .648; and in all experiments in general, the cubic quantity discharged in the observed time is to be reduced to the quantity per second, by dividing the former by the time expressed in seconds; and this, the actual discharge, being divided by the result of the formula expressed in numbers, gives the coefficient by which the formula must be affected to make its results coincide with actual experimental results.

Mr. T. E. Blackwell used in his first set a gauging tank of a capacity of 444.39 cubic feet: we find with an overfall 3 feet long, and a head of .083′ feet, that in 757 seconds the discharge is 137.91 cubic feet, i.e. $\frac{137.91}{757} = .182$ cubic feet per second, now

$$\frac{2}{3} \cdot 3 \times .083 \sqrt{.083} \times 8.024 = .3836 \text{ cubic feet},$$

and .3836 : .182 : : 1 : .4744 = m. In the Table, § 73, these data give $m = .466$; but this is the average of three experiments, of which the above is one. With an overfall of 10 feet long, and a head of 1 foot, we have the discharge equal to 442.29 cubic feet in 15.5 seconds, i. e. $\frac{442\ 29}{15.5} = 28.535$ cubic feet per second, and $\frac{2}{3} \cdot 10 \cdot 1 \cdot \sqrt{1} \cdot 8.024 = 53\ 436$; therefore

$$\frac{28.535}{53\ 436} = .534 = m.$$

82. Mr. Beardmore, in his " Hydraulic Tables," has used the formula—

$$(a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad Q = 214 \sqrt{H^3},$$

in constructing his Table for the "Discharge of Weirs or Overfalls." This formula is very nearly identical with that in § 64, second case; for as this (a) gives the discharge, not for any length, l, but for one foot in length only, and per minute instead of per second, as all the formulæ given in the present work, we must, to compare them, divide (a) by 60 and multiply by l: hence, as $214 \div 60 = 3.566$, we have—

$$Q = 3.566\, lH \sqrt{H} \; \left(= \frac{2}{3}\, m\, lH \sqrt{2gH} \right),$$

and consequently as—

$$\frac{2}{3} \times \sqrt{2g} \times m = 3.566, \text{ we have } m = \frac{3 \times 3.566}{2\sqrt{2g}} = 0.66654;$$

hence, we may write (a) thus,—

$$Q = [illegible] \times l \times 60 \times 3.566\, H \sqrt{H} \quad = [illegible]\, l\, 214\, H\sqrt{}$$

for any length l, and per second, and not per foot of length of overfall, per minute.

This author also remarks, "that the constant 214 is liable to some variation under unfavourable circumstances: for instance, when the weir is formed of a number of short bays, divided by vertical beams, grooved for sliding down the horizontal waste-boards to regulate the surface-level of top water. In these cases, the water passing the edges assumes the *venâ contractâ* form in each bay, and, consequently, the total width, L, of the opening should be reduced to obtain the true quantity of water passing. These, and other causes which may render the observations liable to error, must be treated with judgment, according to circumstances." . . . . "The best way of gauging for weirs is to have a post with a smooth head, level with the edge of the waste-board or sill; *to be driven firmly* in some part of the pond above the weir which has still water. A common rule can then be used for ascertaining the depth, or a gauge, showing at sight the depth of water passing over, may be nailed with its zero at the level of the sill of the weir. Among the conditions essential to a correct result are the absence of wind and current, a good thin-edged waste-board, the water having a free fall, and a weir not so long in proportion to the width above it as to wire-draw the stream; for in this case the water will arrive at the weir with an initial velocity due to a fall, which is not estimated in the gauging, and the result will in all probability be too small."

CHAPTER II.

FLOW OF WATER UNDER A VARIABLE HEAD.

83. *Flow of Water when the level is variable upon one or both faces of the orifice of discharge.*—When a reservoir, instead of being maintained constantly full, as we have supposed it to be hitherto, receives no supply, or receives less than it discharges through an orifice in the bottom, the fluid surface gradually falls, and the tank or reservoir is at length emptied. The laws of the discharge are in this case different from those which have been stated in the first chapter, and the questions to be resolved are of a different character.

The vessels may be either prismatic—that is, of identical sections at every height of the surface—or having sides sloping at some known inclination.

84. *Ratio between the velocities at the orifice and in the vessel.*—Let us suppose that the fluid contained in a prismatic vessel be divided into extremely small horizontal sections, and that they descend parallel to each other, the particles of the fluid in each of the sections must then have the same velocity. This is the hypothesis of the parallelism of the horizontal sections, admitted, and perhaps too much extended, by many hydraulicians.

Let v be the velocity of the particles in the vessel; V that which they have at the orifice; A the horizontal section of the reservoir or vessel containing the water; S, or rather mS, that of the orifice; m being the coefficient of contraction, the volume of water which flows out in the indefinitely small time τ will be expressed by $mSV\tau$. During this same time the surface of the water will descend by a quantity $v\tau$, and the corresponding value of the volume of water will be $Av\tau = mSV\tau$, or $v : V :: mS : A$, giving an example of that hydraulic axiom mentioned § 16, p. 10,—namely, that the velocities are in the inverse ratio of the sections.

85. *Head due to the velocity of the water at its point of discharge.*—The velocity V of the issuing fluid does not now maintain the same constant rate. It is uniform but for a given instant; for, besides being due to the actual head at the given instant, the velocity V, is a consequence of the velocity v acquired during the descent of the parallel sections above mentioned: the two velocities acting in the same direction, from above downwards, their resultant is equal to their sum. Thus, if

$$H' \left(= \frac{V^2}{2g}\right)$$

be the generating height due to the velocity which the water has at its point of discharge, H always representing the actual head in the vessel, we shall have—

$$H' = H + \frac{v^2}{2g} = H + \frac{V^2}{2g} \cdot \frac{m^2 S^2}{A^2} = H + H' \frac{m^2 S^2}{A^2},$$

from whence we may deduce—

$$H' = \frac{H}{1 - \frac{m^2 S^2}{A^2}} = H \cdot \frac{A^2}{A^2 - m^2 S^2}.$$

When mS is small compared with A, as is generally the case, $m^2 S^2$ will be inappreciable with regard to A^2, and may be neglected; in which case, $H' = H$, that is to say, the velocity of issue at any given instant, is that due to the actual height of the water in the vessel at that same moment. In this chapter it is always assumed to be so, although the hypothesis of the parallelism of the horizontal sections, however admissible in their descent, does not hold good when they have arrived at the sphere of action of the orifice, the circumstances of the movement of the molecules of the fluid become then very complicated, and are indeed entirely unknown.

86. *Nature of the motion.*—Let M (Fig. 26) represent a vessel of water filled up to AB; let us divide the height from B to the orifice D into a great number of equal parts, Ba, ab, bc, &c. Suppose, then, that a body, P, were impelled from below upwards with a velocity such that it rises to the point H, PH being equal to DB, and let us divide PH into the same number of equal parts.

In proportion as the body rises, its velocity will diminish, in such manner that when it shall have arrived successively at the points a', b', c', the velocities will be respectively $\sqrt{\text{R}a'}$, $\sqrt{\text{R}b'}$, $\sqrt{\text{R}c'}$ . . . o, as is shown in works on the Elements of Mechanics. Recurring to the fluid contained in the vessel M, in proportion as it flows out, the surface AB is lowered; and when it shall have successively reached the points a, b, c, the respective velocities of the issuing water will be (§ 85) as $\sqrt{\text{D}a}$, $\sqrt{\text{D}b}$, $\sqrt{\text{D}c}$ . . . o, or, according to the construction, as their equals, $\sqrt{\text{H}a'}$, $\sqrt{\text{H}b'}$, $\sqrt{\text{H}c'}$ . . . o; so that, in proportion as the vessel is emptied, the velocity of the discharge will decrease down to zero, following the same law as

the velocity of the body impelled from below upwards, each being an example of a uniformly retarded motion; consequently, the discharge also will be governed by the same law. It will be the same, also, in the descent of the surface of the water in the vessel, which will be uniformly retarded, its velocity being in a constant ratio to that at the orifice,—namely, as the section of the orifice to the area of the surface of the water.

87. *Volume discharged.*—According to the laws of a uniformly retarded motion, when a body, starting with a certain velocity, loses it gradually until it is reduced in zero, it only describes one-half the space it would have traversed in the same time if it had moved uniformly with the velocity with which it commenced the motion. Now the volume of water which flows out from any vessel until it is all discharged may be regarded as a prism, whose base is the orifice, and height the space which the first issuing particles would describe, with a uniformly retarded motion identical with that by which the discharge takes place; but if the same particles had always preserved their initial velocity (which is that due to the primary charge), the space described in the same time, or the height of the prism, and, consequently, the volume of water discharged, would have been doubled. Hence this theorem:—*The volume of water which passes through an orifice at the bottom of a prismatic vessel, receiving no supply, and therefore becoming empty, is only one-half of that which would be given during the time of complete discharge, if the flow had taken place under a constant charge equal to the primary.*

88. *Time which is required to empty a vessel.*—Let H be this charge; A the horizontal section of the vessel; T the time which it may require to be completely discharged. The volume of water discharged during this time—that is to say, all the water the vessel contains (above the orifice)—is $A \times H$. The volume, according to the theorem above, which would have been discharged in that time under the constant charge H, would have been $2A \times H$. This same volume, or the discharge during the time T, is also equal to $mST\sqrt{2gH}$. Equating these two values, we have—

$$2AH = m\,ST\sqrt{2gH},$$

and solving for T, we have—

$$2 \cdot \frac{AH}{mS\sqrt{2gH}} = T,$$

and dividing above and below by $\sqrt{H}$, we have finally—

$$2 \times \frac{A\sqrt{H}}{mS\sqrt{2g}} = T.$$

If we represent by T' the time which the volume AH would take to flow out under the constant head H, we should have had (§ 14)—

$$AH = mST'\sqrt{2gH}, \text{ or } T' = \frac{A\sqrt{H}}{mS\sqrt{2g}},$$

consequently, $T = 2T'$; that is to say, *the time which a prismatic vessel takes to be completely discharged is double that in which the same volume would flow out, if the head had remained constantly the same as it was at the commencement of the discharge.*

89. *Time which the surface of the water takes to descend a given quantity.*—Let t be the time sought in which the level descends the given vertical depth a: now the time in which the whole volume would be discharged is (§ 88)—

$$\frac{2A\sqrt{H}}{mS\sqrt{2g}};$$

the head at the commencement being H; and putting—

$$H - a = h$$

for the head at the end of t, we have the time in which the volume hA would be entirely discharged, equal to—

$$\frac{2A\sqrt{h}}{mS\sqrt{2g}}.$$

Now, the time t, that in which the surface descends a height equal to a, is evidently the difference between the two expressions given above, that is—

$$(a) \quad \ldots \quad t = \frac{2A\sqrt{H}}{mS\sqrt{2g}} - \frac{2A\sqrt{h}}{mS\sqrt{2g}} = \frac{2A}{mS\sqrt{2g}}(\sqrt{H} - \sqrt{h}).$$

90. *Volume discharged in a given time.*—The above expression for the time which the water requires to descend any given height, by a simple transformation gives both the value of a, and also the volume of water discharged during the given time: thus we have from (a)—

$$\frac{tmS\sqrt{2g}}{2A} = \sqrt{H} - \sqrt{h}$$

and—

$$\frac{tmS\sqrt{2g}}{2A} + \sqrt{h} = \sqrt{H};$$

squaring both sides—

$$\left(\frac{tmS\sqrt{2g}}{2A}\right)^2 + 2\,\frac{tmS\sqrt{2g}}{2A}\,.\,\sqrt{h} + h = H;$$

hence—

$$\frac{tmS\sqrt{2g}}{A} \times \left(\frac{tmS\sqrt{2g}}{4A} + \sqrt{h}\right) = H - h.$$

Hence, multiplying both sides by A, we have the discharge Q' for the given time—

$$Q' = (H - h)\,A = tmS\sqrt{2g}\left(\sqrt{H} - \frac{tmS\sqrt{2g}}{4A}\right).$$

91. *Mean hydraulic charge.*—A prismatic vessel (Fig. 27), receiving no supply, discharges water through an orifice, S, during the time T seconds, having at the commencement the head H, and at the end that represented by h; required the mean hydraulic charge H', by which, *ceteris paribus*, the same quantity of water would have been discharged: we have (§ 14)—

$$(b) \; . \; . \; . \; . \; . \; Q' = mS\sqrt{2g}\,T\sqrt{H'} = (H - h)\,A;$$

also (§ 89 a)—

$$T = \frac{2A}{mS\sqrt{2g}}\left(\sqrt{H} - \sqrt{h}\right);$$

substituting this value of T in (b), we have—

$$Q' = mS\sqrt{2g} \times \frac{2A}{mS\sqrt{2g}}\left(\sqrt{H} - \sqrt{h}\right) \times \sqrt{H'} = (H - h)\,A$$

Clearing of fractions, and dividing, we have—

$$\sqrt{H'} = \frac{H - h}{2\left(\sqrt{H} - \sqrt{h}\right)}, \text{ or } H' = \left(\frac{H - h}{2\left(\sqrt{H} - \sqrt{h}\right)}\right)^2.$$

COR.—If $h = 0$, then $H' = \left(\frac{\sqrt{H}}{2}\right)^2 = \frac{H}{4}$.

This proposition gives a method of determining by experiment the coefficient of contraction at an orifice, and has been used by Dr. Young for that purpose (Transactions, Royal Irish Academy, vol. ii., p. 81, &c.). *Vide* EXAMPLES to Chap. II.

92. *Case in which the basin receives a constant stream during the time it is discharging.*—Let us suppose that a prismatic basin receives a constant supply (less than that discharged), and that we have to determine, in these circumstances, the time that the surface will require to fall a given height. Continuing the same letters in the same significations, let us, in addition, call the volume of water arriving at the basin in one second q, and let x be the height which the water descends in the given time t, and dx the height which the surface descends in the indefinitely small time dt, Adx will then express the volume of water which flows out during dt, if the basin received no supply; but as it does receive q in one second, and consequently qdt in dt, the volume of water actually discharged will be $Adx + qdt$. This same volume; from the formula for the discharge of water through orifices (§ 14), is expressed by $mSdt\sqrt{2g(H-x)}$; we shall therefore have—

$$(a) \quad . \quad . \quad . \quad Adx + qdt = mSdt\sqrt{2g(H-x)};$$

and if we make $H - x = h$, and therefore $-dx = dh$, we have—

$$(b) \quad . \quad . \quad . \quad . \quad qdt - Adh = mSdt\sqrt{2g}\sqrt{h},$$

which gives—

$$(c) \quad . \quad . \quad . \quad . \quad dt = \frac{-Adh}{mS\sqrt{2g}\sqrt{h} - q}.$$

In order to integrate this equation, we may put—

$$(d) \quad . \quad . \quad . \quad . \quad mS\sqrt{2g}\sqrt{h} - q = y, \text{ and thus—}$$

$$dt = \frac{-A}{m^2 S^2 g}\left(dy + q\frac{dy}{y}\right),$$

the integral of which is—

$$t = -\frac{A}{m^2 S^2 g}(y + q \text{ hyp. log.} y) + C.$$

Determining the value of C for the commencement of the motion, when $t = 0$ and $x = 0$, and H also being equal to h, we have, substituting for y its value above,—

C equal to—

$$\frac{2A}{(mS\sqrt{2g})^2}\left(mS\sqrt{2g}\sqrt{H}-q+q\,\text{hyp.log.}\,mS\sqrt{2g}\sqrt{H}-q\right).$$

Hence t is equal to—

$$\frac{2A}{(mS\sqrt{2g})^2}\left\{mS\sqrt{2g}(\sqrt{H}-\sqrt{h})+q\,\text{hyp.log.}\frac{mS\sqrt{2gH}-q}{mS\sqrt{2gh}-q}\right\}.$$

It is evident from this expression that when $q=0$, that is, when no supply is flowing in, it becomes identical with that in § 89.

If we had to determine the height which the level of the water would descend in a given time, the question would be reduced to this other—namely, to find the charge h at the end of this time, and subtract it from H, the head, at the commencement of the discharge. To obtain h we must substitute successive values of it; i.e. of $(H-x)$, in the equation given above, and thus tentatively determine that which satisfies the equation.

93. *Case when the water is discharged over a weir.*—In the case when the water issues from the basin by an overfall, supposing that it receive no supply, we shall have, from what has been laid down in § 46 and § 55,

$$Adx = \frac{2}{3}\,ml\,(H-x)\,dt\,\sqrt{2g}\,\sqrt{H-x},$$

whence, by a method analogous to that which has been used above, we have—

$$t = \frac{3A}{ml\sqrt{2g}}\left(\frac{1}{\sqrt{h}}-\frac{1}{\sqrt{H}}\right).$$

94. *Reservoirs not being prismatic.*—We have hitherto considered only the particular case of prismatic basins or reservoirs: the determination of the time of discharge for any other form is much more complicated, and is even impossible in most cases which present themselves. The fundamental equation is, however, always—

$$Adx = mSdt\,\sqrt{2g\,(H-x)},$$

from whence we have—

$$dt = \frac{Adx}{mS\sqrt{2g\,(H-x)}}.$$

But here A is variable, and we must, in order to integrate, express A in terms of x, which can only be effected when we know the law by which A decreases, and in the cases where the basin itself is a solid of revolution, whose generatrix is known. In every other case it will be necessary to proceed by approximations and by parts. To this end, we must divide the basin or reservoir into horizontal sections of small depth. Each of these may be taken as prismatic, and we can determine the time it takes to be discharged by the aid of the formulæ given above. The sum of these partial times will give the time that the surface of the water takes to descend a height equal to the sum of the heights of the prisms.

95. *Flow of water when it is discharged from one reservoir into another.*

1st. In the case when the orifice is covered with water on both faces or sides, the levels remaining constant, the quantity discharged is then the same as if it had taken place into the open air under a charge $H-h$, equal to the difference of the charges upon each face; thus we have, representing by Q the discharge per second,—

$$Q = mS\sqrt{2g(H-h)}.$$

2ndly. Let us suppose that the level remains constant in the upper basin, and that the lower of a given area receives it without any loss, required the time which must elapse before it has reached the level of the upper basin or a given height. This problem is the exact inverse of that in § 89, in which the discharge took place into the air, and the surface of the water above the orifice descended with a motion uniformly retarded. In the present case, the surface of the basin, urged from below upwards with a force measured by the difference of the level between the two basins,—which decreases in the same proportion as the charge decreased in the former case,—rises with a motion uniformly retarded, and it will require the same time to traverse the same space under these similar pressures.

Let H represent the pressure or charge AC (Fig. 28) at the commencement, and h the charge AD at the end of the time t, A the horizontal section of the vessel being filled, and S and m as before,—we shall have, for filling up to DE,

$$t = \frac{2A}{mS\sqrt{2g}}\left(\sqrt{H} - \sqrt{h}\right),$$

and for that of filling up to AF we shall have—

$$T = \frac{2A}{mS\sqrt{2g}}\sqrt{H},$$

These latter formulæ are of some importance: they serve to determine the time in which canal locks, &c., may be filled, and to assign the area of sluice-way required to fill in a given time.

96. 3rdly. *The level of the water being variable in each vessel* —We come now to the third case that can arise, namely, when two reservoirs of different level communicate with each other, each being limited in area and receiving no supply, and thus one surface descends as the other rises. Such is the case of the two basins K and L (Fig. 29), communicating by a wide pipe EF, provided with a sluice-door or cock at G. Before the opening of this sluice-door the level of the water is at AB in the first reservoir, and CO in the second. At the end of a certain time after the opening of the communication it has descended to MN in the first, and has ascended to PQ in the second. It is required to determine the relation between these two heights at a given time, or *vice versâ*, from the given difference in the respective heights, to determine the time corresponding to a given discharge.

Let t equal this time, BE $= H$, CF $= h$, NE $= x$, PF $= y$, A = horizontal section of the first vessel, and B = that of the second, s = section of the pipe of communication: in the coefficient m we must include the resistance of the water passing through this pipe. Whilst the water has risen in the second basin by the quantity dy, during the instant dt, it will have lowered in the other by dx; and remembering that x diminishes while y and t increase, we have $Adx = -Bdy$ and (§ 14)

$$(a) \quad . \quad . \quad . \quad . \quad A dx = -ms\sqrt{2g\,(x-y)}\,.\,dt,$$

from whence—

$$(b) \quad . \quad . \quad . \quad . \quad . \quad dt = -\frac{Adx}{ms\sqrt{2g\,(x-y)}}.$$

The first equation being integrated, remembering that when $x = H$, $y = h$, becomes—

$$(c) \quad . \quad . \quad . \quad . \quad . \quad Ax + By = AH + Bh;$$

solving for y we have—

$$y = \frac{A\,(H-x)}{B} + h;$$

and substituting this value of y in the preceding equation (b), integrating and observing that $x = H$ when $t = 0$, we have—

$$t = \frac{2A\sqrt{B}}{mS\sqrt{2g}\,(A+B)}\left\{\sqrt{B\,(H-h)} - \sqrt{(A+B)\,x - AH - Bh}\right\}.$$

If it were required to find the time in which the two surfaces would be at the same level, we should have from (c)—

$$x = y = \frac{AH + Bh}{A + B};$$

and this value of x being substituted in the above expression for t, will give—

$$(d) \quad . \quad . \quad . \quad . \quad . \quad . \quad t = \frac{2AB\sqrt{H - h}}{mS\sqrt{2g}(A + B)}.$$

From whence it is evident that for the same value of $(H - h)$ t is the same whether A be the horizontal section of the basin that lowers, and B that of the other, whose surface rises, or *vice versâ*, B that which falls, and A that which rises.

EXAMPLES AND PRACTICAL APPLICATIONS.

CHAPTERS I. AND II.

97. *Questions solved by means of the formula* $m S \sqrt{2 g H} = Q$, *the charge on the centre being represented by* H.—In order to obtain a comparative view of the effects resulting from the use of the different coefficients for the discharge through orifices, given in §§ 18 to 43, to which we first confine our attention, let us take a circular orifice of 0.25 ft. in diameter, the area S being therefore $0.25^2 \times .7854 = .04909$ sq. ft., and determine:—First, the discharge through it in some given time, as 40 minutes, with a constant charge of, suppose, 9 ft. above the centre of the orifice; and, Secondly, with the same orifice and charge, seek the different intervals of time required to discharge a given volume of water, as 1000 cubic feet. As the charge is so great compared with the diameter in the above data, we may use the formula (§ 14)—

$$Q' = m S T \sqrt{2g} \sqrt{H},$$

in which H is the charge on the centre. In the first case mentioned above we calculate the value of—

$$m S T \sqrt{2g} \sqrt{H},$$

which becomes $.04909 \times 40 \times 60 \times 8.024 \times 3 = 2836.067$ cb. ft., and multiply it by the several values of m, as is done below. For the second case we have—

$$\frac{Q'}{S \sqrt{2g} \sqrt{H}} \times \frac{1}{m} = T;$$

the value of the first factor of the left-hand side is—

$$\frac{1000}{.04909 \times 8.024 \times 3} = \frac{1000}{1.1817} = 846.24 \text{ cb. ft.},$$

which must be divided also by the successive values of m to obtain T, the time required to discharge the given quantity.

	Value of Q in 40 min. $m \times 2836.067$.	Value of T, or 846.24 cb. ft. $\div$ m. mm.	sec.
(1) m (§ 28) = 0.50	1418 cb. ft.	28	12 internal tube.
(2) m (§ 18) = 0.62	1758	22	45 thin plate.
(3) m (§ 34) = 0.82	2325	17	12 cylindrical adjutage
(4) m (§ 40) = 0.95	2694	14	51 conical converging adjutages.
(5) m (§ 18) = 1.00	2836	14	6 form of *vena contracta* and conl converging.
(6) m (§ 43) = 1.46	4140	9	39 conical diverging adjutages.

(II.) Required the discharge in six minutes, through a rectangular sluice 3 ft. by 1 ft., the side 3 ft. long being horizontal, the depth to the sill from the surface being 7 ft., and m being equal to 0.62.

Here $0.62 \times 3 \text{ sq. ft.} \times 8.024\sqrt{6.5} = Q$,

and $\sqrt{6.5} = 2.5495$ may be taken equal to 2.55,

hence $Q = 38.06$ cb. ft. per sec.,

and $Q' = 6 \times 60 \times 38.06 = 13701.6$ cubic feet.

(III.) A reservoir having at full water a depth of 40 feet over the centre of the discharging sluice, whose area is 2 ft. horizontal by 1.5 ft. vertical when fully opened:—Required the discharge at that depth, and also when the water has sunk to the heads, 30 ft., 20 ft., and 10 ft., the value of m being taken at 0.62 in each case,—we have—

$S = 1.5 \times 2 = 3$ sq. ft., and $\sqrt{40}$, $\sqrt{30}$, $\sqrt{20}$, and $\sqrt{10}$, being respectively 6.324, 5.477, 4.472, 3.162. We must multiply these numbers successively by—

$$0.62 \times 8.024 \times 3 = 14.92464,$$

which is the same in each. Hence, for 40 ft. head the discharge is 94.384 per sec.; for 30 ft., 81.742 cb. ft.; for 20 ft., 66.743 cb. ft.; and for 10 ft., 47.192, or the half of that for 40 ft.; 3.162 being necessarily half 6.324, as they are the roots of numbers in the ratio of 1 to 4. This question points out the fact that leakages of sluices in lock-gates, &c., increase far less rapidly than the head, being, in fact, as the square roots of the charges. (*Vide* Smeaton's Reports, vol. i., pp. 196-9.)

(IV.) What is the discharge through a circular pipe 4 ft. diameter in the embankment of a reservoir, the head upon the centre being 90 ft., m being taken equal to 0.60? In this case—

$$S = (4)^2 \times 0.7854 = 12.5664 \text{ and } \sqrt{90} = 9.487,$$

hence

$$0.6 \times 12.5664 \times 8.024 \times 9.487 = 573.9 \text{ cb. ft. per sec.}$$

(V.) A rectangular sluice, sides 4 ft. horizontal and 3 ft. vertical, having a charge of 20 ft. on the centre, is raised 1.5 ft.: required the discharge per sec., and also when fully opened. We have the value of S in the first instance one-half that in the second, but the heads are 20.75 ft. and 20 ft. respectively; and taking $m = 0.62$, we have first—

$$0.62 \times 8.024 \times 6 \times \sqrt{20.75}\ (=4.5552) = 135.97 \text{ cb. ft.};$$

and secondly,

$$0.62 \times 8.024 \times 12 \times \sqrt{20}\,(=4.472) = 266.97 \text{ cb. ft.}$$

The double of the former would be 271.94 cb. ft.

(VI.) In cities in which water is supplied at high pressure, and constant service, it is sometimes usual to give the water to manufactories and works through a very small orifice, perforated in a disc, which is closed up secure from any possibility of unfair interference. Calculate the discharge through an orifice .0089 in diameter for 24 hours, the head being 129 ft. and m equal to 0.62; we have—

$$\log. m + \log. S + \tfrac{1}{2}\log.(2g) + \tfrac{1}{2}\log. H + \log. 86400'' = \log. Q',$$

the log. of S being 2 log. .0089 + log. .7854,—we have thus—

$$Q' = 303.655 \text{ cb. ft.}$$

(VII.) Suppose the pressure on the mains to be 150 ft. of water, and the diameter of the orifice .02 ft.: required the quantity delivered in 24 hours, the coefficient of discharge being 0.62. The $\sqrt{150}$ being equal to 12.247, and—

$$S = (.02)^2 \times .7854 = .00031416,$$

we have—

$$Q = 24^{h} \times 3600'' \times 0.62 \times .00031416 \times 8.024 \times 12.247$$
$$= 1653.7 \text{ cb. ft.}$$

(VIII.) What must be the diameter of the orifice to give 600 cb. ft. per diem, the head on the main being 100 feet?

Here

$$S = \frac{600\,(=Q)}{24 \times 3600 \times 0.62 \times 8.024 \times 10} = \frac{600}{4298300} = 0.0001396 \text{ sq. ft.}$$

and as $S = d^2 \times .7854$, we have—

$$d = \sqrt{\frac{.0001396}{.7854}} = \sqrt{.0000177744} = .004216 \text{ ft.}$$

As the exact adjustment of this diameter would be nearly impossible, the process is somewhat tentative.

(IX.) In the sluices constructed in tidal harbours for scouring away at low water the silt that generally accumulates in them, we obtain examples on a very large scale of the discharge of water through orifices.

This simple remedy for a defect that had rendered nearly useless some of the most important tidal harbours on the coast of England, not having the advantage of any sufficient natural streams to keep them open, was introduced by the great Smeaton from his personal observation of the practice in the Low Countries (*vide* Reports, vol. ii. p. 202–209). A bank thrown across

some part covered at high tide impounded the water allowed to enter during the rise of the tide, and which at low water is discharged very rapidly through sluices constructed in this embankment, the sills of which are at low water of springs, or as low as possible.

The practice subsequently fell into disrepute, as it was found that the area of the back-water was itself soon silted up; but the same engineer adopted the simple and efficient remedy of dividing the back water into two separate areas by a second bank at or about perpendicular to the first mentioned, and by occasionally using one of these to cleanse the other, they were both, as well as the harbour itself, kept clear. Ramsgate and Dover are well-known examples (*vide* Smeaton's Reports and Sir J. Rennie on Harbours); from which last-mentioned work we take an example from the description of Hartlepool Harbour, on the coast of Durham.

Each sluice was 3 feet wide and 6.33 feet high, having a charge estimated at 10 ft. on the average. From the detailed plans of these works given by Sir J. Rennie, we may consider the coefficient 0.600 applicable; hence—

$$0.600 \times 3 \times 6.33 \times 8.024 \times \sqrt{10} = 289.14 \text{ cb. ft. per sec.}$$

is the discharge for each sluice; and as it is also stated, that the total area of the scouring sluices was 366 sq. ft., of which 24 sq. ft. was given by four sluices, each 3 ft. by 2, in the lock-gates, which communicated with the back-water or slake, we have 342 sq. ft. left for those through the embankment; and each of these being $3 \times 6.33 = 19$ sq. ft., we have their number 18, i.e. $342 \div 19$; and the discharge for one being 289.14 cb. ft., the total discharge is $18 \times 289.14 = 5204.52$ cb. ft. per sec., or 312271 cb. ft. per minute. Now the back-water containing 15420000 cb. ft., it could be discharged in about 50 minutes ($15420000 \div 312271$). It is essential that the back-water should be discharged rapidly before the rising tide diminishes the force of the artificial scouring action.

(x.) The widely different statements as to the efficiency of hydraulic engines as prime movers, some being asserted to give as high as 80, and others 60 per cent. of the power used, may in some cases, perhaps, be traced to a false estimate of the quantity of water discharged upon the prime mover used; for unless this be actually gauged, it must be calculated, and some coefficient used. In the case of undershot wheels with sloping sluices, as in Poncelet wheels, the bottom and sides being in continuation of the channel of supply, the coefficient is

0.74 when the sluice is inclined 1 base to 2 height, and 0.80 when 1 to 1 (Claudel, Aide Memoire, p. 78, § 100). If we had taken it 0.62, and with a six feet fall, the sluice being supposed 6 feet wide and raised 1 foot, we have—

$$0.62 \times 6 \times 8.024 \times \sqrt{6} = 73 \text{ cb. ft.};$$

and had the modulus of the machine been calculated to be 80 per cent. from this discharge, it would, in reality, be but 67 per cent. found by the proportion 0.74 : 0.62 : : 80 : 67.

(XI.) If the modulus of a water-wheel be estimated at 88 per cent., with a coefficient of discharge of 0.65, the wheel being 7 feet broad, and the sluice, which slopes at 1 to 1, raised 0.75, the head being 5.5 ft.: required the true modulus.

Here $0.65 \times 0.75 \times 7 \times 8.024 \times \sqrt{5.5} = Q = 64.2$ cb. ft.; hence (as $.80 - .65 = .15$), $Q(1 + 0.15) : 64.2 : : 88 : 76.56$ per cent., the true discharge on the wheel being 73.8, i. e. $64.2 \times .15$.

(XII.) *Determination of the relative level in two vessels communicating by a submerged orifice.*—Let a vessel or cistern, A (Fig. 30), receive a constant supply of water, and discharge it into another vessel, B, through the submerged orifice a, which finally discharges into the air freely: the orifice at b is 1.0 feet horizontal by 0.2 feet vertical, and the charge H upon the centre 1.25, the coefficient being taken equal to 0.62; hence we find the discharge $0.62 \times 8.024 \times 0.2 \times 1.118 = 1.11238$ cubic feet per second, which must be equal to the constant supply received by A, and transmitted by it through the orifice a to B. Now this orifice is 0.8 feet by 0.1 foot, being a sluice 0.8 feet wide, and raised 0.1 feet,—capable, however, of being raised to 0.5 feet; and hence the charge upon it, which must be reckoned from the surface of B, is equal to $\left(\dfrac{1.11238}{0.62 \times 8.024 \times .08}\right)^2 = 7.806$ feet; and, as we should expect, the square roots of these charges are inversely proportional to the areas of the orifices; that is, 2.794 : 1.118 : : 0.2 : 0.08. Hence, if we suppose the constant supply to be so increased as to raise the surface of the water in A one foot above its level in the last case, we may determine the corresponding rise in B, and also the additional quantity that has been supplied. The total vertical height above the centre of b is now $1.25 + 7.806 + 1 = 10.056$, which number has to be divided into parts whose square roots have the ratio 0.2 to 0.08, that is, of the areas of the orifices. Hence $(0.2)^2 : (.08)^2$ being as 4.00 to 0.64, we have—

$$4.64 : .64 : : 10.056 : \frac{10.056 \times .64}{4.64} = 1.387$$

for the one part, i.e., the surface of B above b, and 10.056 $-$ 1.387 = 8.669 for the other, that is, of the surface of A above that of B, and the quantity received in A is now 1.172 cubic feet per second. The actual rise of 1 foot in A corresponded, therefore, to one of (1.387 $-$ 1.25 =) .137 feet in B. If we had supposed the surface of A *lowered* 1 foot, then we find that B descends 0.1388 ft., and the constant supply is now 1.05 cubic feet per second. Hence the total range of B is only 0.277 feet for the corresponding change of 2 feet in A; a result which points out the value of the second or lower regulating chamber in insuring a nearly constant head on the outer orifice b, and consequently a discharge also nearly constant, though the really effective head in A may vary considerably: a very important application is described in Examples XXIII. XXIV., p. 83.

(XIII.) The time of filling a lock on a navigable canal (Fig. 31) consists of two distinct intervals: one, the time of filling up to the centre of the sluices; the second, that of raising the surface up to the level of the upper reach. The length of a lock being 115.1 ft., and breadth 30.44 ft., the horizontal area is 3503.6 square feet, and the vertical depth from centre of sluice to lower reach 1.0763 feet, the charge being 6.3945 feet; hence, the cubic content of the lower portion, that is, the value of Q', is 3771 cubic feet; the area of the two sluices 2 × 6.766 square feet = 13.532 square feet; and the charge on centre, as above, 6.3945 feet; and the value of m, assumed by D'Aubuisson, is 0.548 ft. From some experiments on the Canal of Languedec it was found that when two sluices were opened in the gates, the discharge was not double that given when only one was used: it was found, in fact, to be about an eighth part less, which reduces m from 0.625 (§ 24) to 0.548. We have therefore—

$$\frac{Q'}{m \,.\, S \,.\, \sqrt{2g} \cdot \sqrt{H}} = T = \frac{3771 \text{ cb. ft.}}{0.548 \times 13.532 \times 8.024 \times \sqrt{6.3945}} = 25''.$$

98. *Determination of the charge necessary to give a certain quantity with a given value of S.*—To determine the head necessary to give a certain discharge, we have but to solve $Q = m\,S\,\sqrt{2g}\,\sqrt{H}$ for H; and hence—

$$\left(\frac{Q}{mS\sqrt{2g}}\right)^2 = H.$$

(XIV.) Required the head necessary to give 7.85283 cb. ft. per sec. through an orifice 0.5 feet square, m being equal to 0.625. Here—

$$\left(\frac{7.85283}{1.24175} = 6.324\right)^2 = 40 \text{ feet},$$

or 2 (log. 7.85283 − log. 1.24175) = log. H; that is—

$$2\,(0.8950245 - 0.0940167) = 1.6020156 = \log.\ 40.$$

If the orifice had been 0.75 feet square, determine the charge necessary to give the same discharge as in the last example, namely, 7.85283 cubic feet per second. Here—

$$\left(\frac{7.85283}{2.821} = 2.7837\right)^2,$$

and $\quad 2 \log.\ 2.7837 = 2\,(.4446224) = 0.8892448,$

giving $\quad H = 7.748984$ feet.

What additional head would each orifice require to discharge 10 cubic feet per second, the coefficient remaining 0.625?

Here $7.85283 : 10 :: \sqrt{40} : \frac{63.24}{7.85283} = \log. 63.24 - \log. 7.8528$ $(= 1.8009919 - 0.8950245) = 0.9059674 = \log.\ 8.053$, which is the square root of the charge required, whose value is therefore 64.85 ft., and deducting 40, we have the increase of head equal to 24.85 feet.

And, $7.85283 : 10 :: \sqrt{7.748984} : \frac{27.837}{7.85283} = \log. 27.837$ $- \log. 7.85283\ (= 1.4446224 - 0.8950245) = 0.5495979 = \log.$ of 3.5448, which is the square root of the charge sought, $2 \times .5495979 = 1.0991958 = \log.$ of 12.575, from which deducting 7.748984, we have the additional head in this equal to 4.826 feet.

(xv.) Calculate the head that is equivalent to the difference between the coefficients 0.600 and 0.950; that is, having the discharge under certain data, with $m = 0.950$: determine what additional head would be required to give the same discharge when $m = 0.600$. Thus, let the charge on the centre be 8.55 feet, the orifice circular and 0.045 feet diameter, and so nearly the form of the *venâ contractâ* that the coefficient rises to 0.950; we have therefore—

$$S = (.045)^2 \times .7854 = .00159, \text{ also } \sqrt{8.55} = 2.924,$$

and $Q = .95 \times .00159 \times 8.024 \times 2.924 = .03544$ cb. ft., and the head necessary to give this discharge with $m = 0.6$ is found (as $mS\sqrt{2g} = 0.6 \times .00159 \times 8.024 = .007655$) by—

$$\left(\frac{Q}{m\ .\ S\sqrt{2g}}\right)^2 = \left(\frac{Q}{.007655}\right)^2 = H\ . \text{ or by logarithms,}$$

$$\left(\frac{.03544}{.007655}\right)^2 = (\bar{2}.5494937 - \bar{3}.8839452) \times 2 = 1.331097$$

$= \log.\ 21.43$ feet.

Thus 21.43 − 8.55 = 12.88 feet is the additional head or pressure required to discharge the same volume of water through the orifice in a thin plate that was discharged with 8.55 feet pressure through an orifice nearly of the true form. Thus, the accelerating force due to this form, when compared with the thin plate, is measured by a pressure equal to more than one-third of the weight of the atmosphere.

99. *Results of the suppression of the contraction on part of the perimeter*, §§ 25 *to* 26.—A sluice 3 feet square, and with a charge on the centre of 12 feet, has, from the thickness of the frame, the contraction suppressed on all sides when fully open; but when partially opened, the contraction exists on the upper edge, that is, against the bottom of the gate, which is formed of a thin plate of metal. Required the discharge when opened 1 foot, 2 feet, and fully opened.

(XVI.) When opened 1 foot high, the total perimeter is 8 ft., and the part on which the contraction is suppressed is 5 ft.: hence—

$$\frac{n}{p} = \frac{5}{8}.$$

Hence, from the formula (§ 26)—

$$m \,.\, S \,.\, \sqrt{2g}\, \sqrt{H}\left(1 + .152\frac{n}{p}\right),$$

m being 0.608, we have for the discharge—

$$0.608 \times 3 \times 8.024 \times \sqrt{13}\left(1 + .152\frac{5}{8}\right) = 57.77 \text{ cb. ft.}$$

per second; the two last factors being 3.605 and 1.095.

When opened 2 feet high the total perimeter is 10 feet, and the contraction suppressed on 7 feet; so that—

$$\frac{n}{p} = \frac{7}{10},$$

and—

$$0.152 \times \frac{7}{10} = 0.106,$$

also $\sqrt{H} = \sqrt{12.5} = 3.5355$. Hence the discharge in this case is $0.608 \times 6 \times 8.024 \times 3.5355 \times 1.106 = 114.45$ cubic feet per second.

When the sluice is fully opened, the total perimeter is 12 feet and $\frac{n}{p} = 1$; so that the discharge is—

$$0.68 \times 8.024 \times 9 \times 3.464 \times 1.152 = 195.9 \text{ cb. ft. per sec.}$$

100. *Questions upon §§ 44 to 52, formulæ* $\frac{2}{3} ml\ H\sqrt{H}\ \sqrt{2g}$ *and* $\frac{2}{3} ml\ \sqrt{2g}\ (H\sqrt{H} - h\sqrt{h}) = Q$. — We derive from Captain Baird Smith's work on "Italian Irrigation" many examples whose solution is given by the formulæ in these sections. It appears in the irrigated districts to be a matter of great importance to determine both a unit of volume of water, and also some means of measuring it so as to regulate the due supply to each proprietor, that, on the one hand, the Government, or individuals having the ownership or management of the canals of irrigation, may not be defrauded; nor, on the other, the proprietors of the land to be irrigated suffer any injustice. Many very different units are found in use at present. In the earlier periods of the construction of these canals a fixed *area* of orifice, opened in the side of the canal, was alone used, without any reference to the head or charge under which the water issued: to this was subsequently added the condition of a fixed charge as well as a fixed area, but without any mechanical arrangements for insuring this constant pressure, which, from the continual variations in the level of the surface of the water in the canal, was absolutely requisite; however, unless the excess or deficiency complained of was more than the eighth part of the total volume specified in the grant, no complaint was admitted. A very simple and beautiful contrivance, founded on the effect of the double cistern on the discharge (§ 97 (XII.)), and described below, is now, however, in use on many of these works of irrigation, which meets this difficulty, sufficiently, at least, for practical purposes.

101. Taking these different measures or units in the order of date, as nearly as possible, it appears that on the Canal of Caluso the unit called *ruote*, or wheel, was defined to be the quantity passing through an opening whose area is 1 foot square,—this foot being equal to 1.6702 feet lineal English,—the upper edge of the outlet being what is locally termed *a fior di acqua*, or level with the surface of the canal or reservoir, the discharge hence taking place under no pressure. The volume discharged by the *ruota* is estimated by the Piedmontese engineers at 12.05 cubic feet English per second; but obvious sources of error and discrepancy arise from the reference being solely to the superficial area: first, on theoretical considerations; and secondly, on practical. For we may evidently have a number of orifices with very different discharges and figures, all fulfilling the condition of a constant area. The

Piedmontese foot being divided into 12 inches, the value of an inch is 1.6702 English inches. The following short Table, limited to whole numbers of inches, will illustrate this point (all in Piedmontese inches):—

Linear dimensions applicable to a Ruote.

	Height H.		Breadth l.		Constant area.	Perimeters.	Ratio of sides.
1.	12	×	12	=	144	48 ...	1 : 1
2.	16	×	9	=	144	50 ...	1 : 0.5625
3.	18	×	8	=	144	52 ...	1 : 0.444′
4.	36	×	4	=	144	80	1 : 0.111′
5.	48	×	3	=	144	102 ...	1 : 0.062
6.	72	×	2	=	144	148 ...	1 : 0.0277′

On examining this Table we see the heights vary from 1 to 6, and the perimeters from 1 to 3, the area remaining constant. Many outlets are instanced, showing that this unit was actually used with a great variety of figure in many grants of water. But, secondly, a practical objection against any measure whose outlet is level with the supplying surface arises from the fact, that this surface is always changing, not only from the variations in the level of the supplying river, but from the changes in the demand for water from the canal by the various land-owners along its length.

(XVII.) Calculate the discharge which would be given by Nos. 1, 3, 5, and 6, of the above Table,—m being taken equal to 0.62, the foot being 1.6702 ft. English, we have $\frac{2}{3} \times .62 \times 8.024 = 3.315$, and $l \times H = 2.789$ sq. ft. common to each; and thus—

For No. 1, we have $3.315 \times 2.789 \times \sqrt{1.6702} = 12.00$ cb. ft.
„ No. 3, „ $3.315 \times 2.789 \times \sqrt{2.505} = 14.63$ „
„ No. 5, „ $3.315 \times 2.789 \times \sqrt{6.6808} = 23.90$ „
„ No. 6, „ $3.315 \times 2.789 \times \sqrt{10.02} = 29.26$ „

Or generally, the discharge varying as $l \, . \, H \, . \, \sqrt{H}$, and $l \, . \, H$ being constant, it is evident that it increases as $\sqrt{H}$; so that, by increasing the depth indefinitely at the expense of the width l, we increase the discharge. Thus, let $H = 16$, the log. of $\frac{2}{3}\, mlH \sqrt{2g}$, that is, of $(3.315 \times 2.789 =)\, 9.2455$ being 0.9659304, we must add to it half the log. of H for the log. of the discharge: half the log. of 16 is 0.6020600, and adding, we have $1.5679904 = \text{log. of } 36{,}982$ cb. ft.

102. *Questions on the formula* $\frac{2}{3}\, ml \sqrt{2g} \, . \, (H\sqrt{H} - h\sqrt{h}) = Q$, *the Italian dimensions being all reduced to English measures.*

(XVIII.) Ignazio Michellotti having determined to modify the mode of measuring a *ruote* introduced by his father, F. D. Mi-

chellotti, which had the upper edge level with the surface of the supplying canal, and was estimated to give a discharge equal to 11.83 cb. ft. per sec., defined the *uncia* or inch of water to be that flowing through an orifice 0.5567 ft. high, 0.41755 ft. wide, and having a pressure on the upper edge of 0.5567 ft. This he supposed would give the twelfth part of 11.832 cb. ft., or 0.986. Calculate its true value: m being 0.600, we have then $H = .5567 + .5567 = 1.1134$ ft., and $l = .41755$ ft.

Ans. 1.02 cb. ft.

(XIX.) The measure used on the Canal Lodi was defined to be 1.12 ft. by 0.12416′ ft. wide, with a charge on the upper edge 0.32 ft., and these dimensions were supposed to give 0.77 cb. ft. per sec. Here $H = 1.12 + .32 = 1.44$ ft., and $l = .12416'$ ft.

Ans. 0.6165 cb. ft. per sec.

(XX.) That used on the canal of Cremona was 1.31816′ ft. high by .131 ft. wide, having a head also .131 ft., and estimated to discharge 0.88 cb. ft. Hence—

.715 c.

$$\tfrac{2}{3}\,0.6 \times .1727\,(1.449\sqrt{1.449} - .131\sqrt{.131})\,8.024 = 0.94 \text{ cb. ft.}$$

(XXI.) That of Crema was 1.276 ft. high, 0.1275 ft. wide, a charge of 0.255 ft.: calculate the discharge.

Ans. 0.7225 cb. ft. per sec.

(XXII.) The Sardinian Civil Code determines the unit in which all grants of water should be expressed thus:—"The measure or *modulo* (Fig. 32) is that quantity of water which, under simple pressure, and with a free fall, issues from a rectangular quadrilateral opening, so placed that two of its sides shall be vertical, having a breadth of 0.6562 ft. (English measure), and a height also of 0.6562 ft. It shall be opened in a thin wall (or plate-*parete*), against which the water stands, with its upper surface perfectly free, at a constant height of 1.3124 ft. ($= 2 \times 0.6562$) above the lower edge of the outlet." It is required to calculate the value of this unit in cubic feet per second. We have therefore $l = .6562$, and H and h, being 1.3124 and .6562 respectively—

$$\tfrac{2}{3} \times .6 \times .6562\,(1.3124\sqrt{1.3124} - .6562\sqrt{.6562})\,8.024$$
$$= 2.046 \text{ cb. ft. per sec.}$$

When grants are made for more than one *module*, the only dimension which varies is the breadth of the outlet, the height and pressure remaining in all cases invariable; two *modules* would have a breadth of outlet of 1.3124 ft., three would have 1.9686 ft., and so on.

103. *Description of a Piedmontese outlet* ("Italian Irrigation," pp. 21, 22, vol. ii.).—"AB (Fig. 33) is the transverse section of the supplying canal; the first part of the measuring apparatus is the sluice, which consists of masonry, side walls, and a gate of wood, working vertically. The dimensions of this primary outlet are not rigidly fixed, its object being merely to admit a larger or smaller supply into the chamber CD. The sluice is established in the bank of the canal, at such point as may be fixed upon by the canal authorities, or most convenient for the land-owner. Its sill is sometimes on the same level as the canal bed, sometimes above it, and very frequently as represented in the diagram. There is a fall in front of the outlet, so as to draw the water towards it. For a length of from 40 to 50 feet from the sluice, the bed of the channel is made perfectly horizontal, paved with masonry or cut stone, the upper surface of which is on the same level as the sill of the sluice. At a distance from the outlet, ranging from 16 to 32 feet, is fixed the partition or slab of stone *cd*, in which the regulating or measuring outlet *ef* is cut, of which the height is fixed at 0.56 ft., while the breadth varies with the number of units or inches, to be given, each inch being represented by 0.42 ft. of breadth. The lower edge of the measuring outlet is ordinarily placed at 0.819 feet above the level of the flooring of the chamber CD. A small return, cut in the inner face of the slab at a height of 0.28 ft. above the upper edge of the outlet, indicates the constant level of the water necessary to insure the established pressure. This height is maintained by the raising or lowering, as may be requisite, of the sluice at the entrance of the chamber.

(XXIII.) Calculate the value of a grant of three inches of water from this structure. We have $H = .56 + .28 = .84$; hence—

2.514 c.ft

$$0.6 \times 0.42 \times (.84\sqrt{.84} - .28\sqrt{.28})\, 8.024 = 1.257 \text{ cb. ft. per sec.}$$

104. *Description of the modulo magistrale of Milan.*—This module, as applied upon the Naviglio Grande, which, in a course of 31 miles from its head on the river Ticino to the city of Milan, distributes 1851 cb. ft. per second, is in its principle identical with that already described (§ 103). For the interesting history of this canal, and the gradual improvements in the management of the grants of water, we refer to "Italian Irrigation," vol. i. pp. 203, 228; vol. ii. pp. 36, 56. The honour of the discovery is due to Soldati, of Milan, about the year 1571, who invented it in answer to an invitation from the magistracy of that city to architects and engineers to design a measuring apparatus.

The unit fixed upon, called the *oncia magistrale*, had the following dimensions (Fig. 34):—Height, 0.655 ft.; breadth, 0.3426′ ft.; with a constant pressure of 0.32944 ft. above the upper edge of the outlet. When one outlet is designed for the discharge of several water-inches, the breadth only varies, in the proportion of 0.3426′ ft. for each additional water-inch, the height and pressure remaining constant, as in Fig. 35, which shows an outlet for six water-inches. The outlet is cut with care in a single slab of stone. To preserve it from being tampered with, an iron rim is fixed upon it, of the exact dimensions corresponding to the discharge. They ought invariably to be cut in a simple plate, with no arrangement of any kind to increase the volume beyond that due to pressure alone. The thickness of the slab varies somewhat with the dimensions of the outlet; but in a rigidly exact *module* this dimension should be fixed in common with all the others. These are the conditions applicable to the measuring outlets, the discharge from which is—

$$0.6 \times .34266'\left(.98444\sqrt{.98444} - .32944\sqrt{.32944}\right) 8.024 = 1.3 \text{ cb. ft.}$$

To illustrate the other arrangements of the *modulo*, the plan and section (Figs. 36 and 37) are given from the same work.

The sluice AB (Fig. 36) is placed on the bank of the canal of supply, with the sill CD (Fig. 37) on the same level as the bottom of this canal. It is formed of two side-walls or cheeks, of good masonry, in brick or stone, with a flooring generally of the latter material. To prevent erosive action, the bed of the canal, for such distance as the force of the current may render necessary, is paved with slabs of stone or boulders, both above and below the head. The sluice-gate is usually made of the same breadth as that of the measuring orifice GH (Fig. 37), while its height is regulated by that of the sluice itself. The sluice-gate or *paratoja* IK (Fig. 37) works in grooves, and is fitted with a rack and lever, by which it can be readily raised or depressed at pleasure. As the surface level of the canals of the Milanese varies comparatively little, the upright of the sluice has a small catch in iron or wood attached to it, by which it is kept at a fixed height, corresponding to the requisite pressure on the original orifice GH (Fig. 37). This little catch is locally termed the *gattello;* and as it is provided with a lock and key, the latter of which is intrusted to the guardian of the canal, the proprietor of the water-course supplied through the *module* is supposed to be restricted to his legitimate supply, and probably is so within reasonable limits, provided always that the guardian is incor-

ruptible. In the rear of the sluice-gate, at the head, is placed the first chamber LM (Figs. 36 and 37), called the *tromba coperta*, or covered chamber. Its length is equal to very nearly 20 feet, and a breadth variable according to the size of the head-sluice, which it exceeds by the fixed quantity of .82 ft. on each side, or 1.64 on the entire breadth. The bottom of the covered chamber DH (Fig. 37) is formed with a slope to the rere, the height H*h* being 1.3125 ft. English: its object is to diminish the velocity with which the water reaches the measuring outlet GH. Further to assist in effecting this object, the perfect *modulo* is provided with a horizontal top of stone slabs or planks, called the *cielo morte*, the under surface of which is at precisely the same height as the water ought to have over the outlet GH, so as to secure the fixed discharge, that is, 0.32944 ft. above the upper edge of GH. It is found that this does reduce the irregular motion of the water, and so tends to secure the great object of the *modulo*, that the discharge take place under simple pressure, and without antecedent velocity. To admit of ready inspection of the height of the water within the covered chamber, the following arrangements are made:—The entrance to the chamber is covered with a stone slab of convenient thickness, shown in section at E (Fig. 37), the lower surface of which is precisely on the same level as the upper edge of the outlet GH. The height of the slope H*h* being 1.3125 ft., and that of the outlet GH being 0.655, the surface of the slab at E should be 1.9483′ ft. above the sill of the head CD. An open groove LD is made in the masonry, large enough to admit a graduated rod or measure; and when the water stands at a height of—

$$(1.9843' + .3275 =)\ 2.3118 \text{ ft.}$$

above the sill at D, it is known that the proper head of pressure exists at GH. As it is found to be greater or less, the sluice is raised or depressed, so as to adjust the pressure to the fixed standard. The slab of stone in which the measuring outlet is cut being fixed at GH (Figs. 36 and 37), immediately in rere of it there is placed the *tromba scòperta*, or open chamber. Its breadth at N (Fig. 36) is two local inches (0.3275 ft. English), greater on each side than that of the measuring outlet, or on both sides .6550 ft. Its total length NO is very nearly 17.75 ft. English. Its side-walls, which are perpendicular, like those of the covered chamber, have a splay outwards, so that the breadth at O is 0.9825 ft. greater than at N, or 1.31 ft. in excess of that of the regulating outlet GH, being the same as that of the covered chamber throughout. To

insure the free run of the water from GH, the flooring of the open chamber has a drop or fall of 0.1633′ft. at H, and an equal quantity distributed uniformly between H and O (Fig. 37). There is therefore a total fall from the under edge of the measuring outlet to the end of the open chamber of 0.3275 ft., or, as the length is 17.72 ft., of 1 in 54. When the water reaches O, it enters the channel of distribution for the use of the consumers: generally the point O, and the bed of the channel, which is carried on at the usual inclination, are upon the same level, but sometimes there is a fall.

105. From the preceding details, it appears that the *modulo magistrale* has a total length of nearly 37.75 ft. English, and a breadth variable according to the quantity of water it is intended to measure. If a single water-inch, for instance, be granted, the breadth of the covered chamber would be 2.12835 ft., and that of the open chamber 1.145835 ft. at its upper, and 2.12835 at its lower extremity. The flooring of the former rises 1.31 ft. to the rere, while that of the latter falls 0.3275 ft. in the same direction. It is essential to the effective operation of the regulating sluice in the *modulo magistrale* that there should be a difference of level between the water in the canal and in the apparatus of at least 0.655 ft.; and as the height of water in the latter must be 2.3118 ft., the depth of water in the canal of supply must necessarily be not less than the sum of these numbers, or 2.945 ft., very nearly 3 ft.

It is curious to reflect that this apparatus was invented empirically by Soldati, in 1571, so many years before the discovery of the Toricellian theorem, which must be placed in the year 1643, when that philosopher showed that the velocity of running water was identical with that of falling bodies, the foundation of all our knowledge of Hydraulics. This is not the only instance in which the practical sagacity of the engineer has anticipated the discoveries of theory.

Two essential objects are fulfilled by the arrangements just detailed:—1st. To maintain on the measuring outlet a constant pressure; and 2nd. To make this pressure as much as possible the sole force influencing the discharge, that is, that the water have no velocity antecedently. The first is secured by the mechanical arrangements at the head,—the sluice, with its rack, lever, &c., and to a certain extent the *cielo morte*. By raising or lowering this sluice the level of the water in the covered chamber is maintained, independent of the variations in the surface of the external canal. The second by the interior arrangements,—the covered chamber with its fixed top, and floor sloping up to the outlet; while the free passage of the

water is secured by the open chamber, with its small fall at the head and continued inclination at the bottom.

106. The differences in the estimates of the quantity of water discharged by the *modulo magistrale*, as given by different Italian engineers, are very remarkable, considering the great attention that has been paid to the theory and practice of Hydraulics in that country. De Regi gives it as 1.42 cb. ft. per sec.; Breschetti states the average result of experiments on the Muzza Canal to give 1.57 cb. ft. per sec.; Mazzeri estimates it as low as 1.21 cb. ft.; Brunacci at 1.46; while the Department of Public Works in Lombardy considers it equal to 1.64 cb. ft. per sec. The extremes, we see, are 1.21 and 1.64 cb. ft. per sec., —a difference of ·43 cb. ft., between a third and fourth of the total discharge. Captain Smith accounts for this great difference by stating—"That the estimate of the Government is founded on the experience of the results on the great canals, where the outlets are almost uniformly of large dimensions" (pp. 222, 223, vol. i.). Now it is certain that, all other circumstances being alike, the quantities of water discharged from large are proportionally greater than those discharged from small outlets. Hence the *oncia magistrale*, as determined by experiments with the former, has a decidedly higher value than when determined by the latter. The cause of this is clear. To give a discharge of, say, six water-inches, the breadth of the outlet is made six times that for one inch, the height and the pressure remaining in both cases the same. The proportion between the sectional area and perimeter of the outlets becomes, however, materially altered, and the influence of the perimeter in effecting the contraction of the vein diminishes gradually as the size of the outlet increases; and in a similar proportion the discharge becomes greater. To elucidate this, it may be remarked, that in an outlet for one *oncia magistrale* the ratio of the section to the perimeter is as 1 to 23.33; for two, as 1 to 16.66; for four, as 1 to 13.33; for eight, as 1 to 11.66; for ten, as 1 to 11.33, or about half what it is for one *oncia;* for twenty *oncia*, as 1 to 10.66, and so on; and there are real differences of discharge due to the variable ratios now given. Very serious pecuniary loss may consequently be the result to the proprietors of the canal or the consumers of the water. It appears (vol. i. pp. 226, 227) that for summer irrigation each cubic foot per second is capable of irrigating 61.8 acres, and that the annual rent of this quantity, summer and winter, is £13 5*s.*; the difference of .43 cb. ft. between the highest and lowest estimate of the discharge of the *modulo magistrale* is worth £5 13*s.*, and would irrigate 26 acres

at the above rate. The recognition of the differences between the discharges of large and small outlets was very early made in Lombardy. In the *module* of Cremona, invented in 1561, no single outlet was allowed to exceed 1.31 high by 3.18 broad, equal to about 12 or 13 water-inches. In the Milanese single outlets have been restricted for nearly three centuries and a half to discharges of from 9 to 12 *once*. In Piedmont they have been more careful, and have there limited single outlets to 6 *once*, which, by general consent, seems to be the most approved size for diminishing to the utmost the error due to the inequality of discharges from large and small openings. For practical purposes, therefore, and taking the mean of the various estimates of the value of the *oncia magistrale* just adverted to, it may be considered as equal to very nearly 1½ cb. ft. per second.

107. Another mode of insuring a constant discharge through an orifice having a charge subject to variation has been brought into use by the late Mr. Thom, an hydraulic engineer of great eminence. It attains this object by mechanical means chiefly. Figs. 38 and 39, taken from the description of the Gorbals Waterworks, near Glasgow (Practical Mechanics' Magazine, August, 1848), will serve to explain this method. In Fig. 40 we have a transverse section of the embankment of a reservoir. L, L, is the level of the surface of the impounded water, and *l*, *l*, that of the receiving basin below, supplied from the reservoir by a pipe through the base of the embankment. Should the quantity drawn off by the town or mill to be supplied increase, then the level of the surface *l*, *l* will descend; and the regulating apparatus should be of such a construction that it may permit a larger quantity to pass through the pipe, and *vice versâ*. Again, if the level of *l*, *l* should remain constant, and, from an increased or diminished supply, that of the reservoir L, L rise or fall, then this apparatus should be so constructed as to alter the orifice of the discharging main pipe that it deliver only that constant quantity carried off from the receiving basin, and needed for the works. Fig. 38 gives a longitudinal section of the detail of the regulator: *d* is a cast-iron cylinder or float attached at top to a chain passing over the fixed wheel *c*, and inclosed by another cylinder of a diameter slightly larger, represented in section at *e*. The other end of this chain is fixed to the bent lever *b*, working freely on a stud carried by two cast-iron brackets screwed to the extremity of the pipe, which passes through the base of the embankment of the reservoir, and terminates in a square mouth-piece, faced to receive a square hinged flap-valve, *a*, which is retained in any

desired position by the lower and shorter arm of the bent lever, which works against the back of the valve by an anti-friction roller at *v*. Now if we suppose the water in the outer cylinder *e*, to stand at the level *s s*, the cast-iron float being partly immersed to a certain depth below this surface, its weight, acting by the chain upon the bent lever *b*, will press against the square flap-valve, and thus partly close the mouth of the main-pipe, restricting the discharge through it to the desired quantity. Suppose, then, that from any circumstances this discharge should become too small, it will then be necessary that the self-acting apparatus should be such as to permit the valve to open, and therefore, also, the cast-iron float to rise, which it will do if the water-level in the outer cylinder be made to rise; for then the cast-iron float becomes specifically lighter, and presses with a less force upon the valve *a*, which immediately yields to the pressure of the water issuing through the discharge-pipe, and thus permits a greater quantity to escape. If, on the other hand, the quantity discharged had been too great, it will be necessary that the cast-iron float descend, and thus press the flap-valve closer upon the square face of the discharge-pipe. This it will do if the water in the outer cylinder be made to fall; for thus the float becomes specifically heavier, and sinks, closing the flap-valve *a*: so that we have to devise such mechanical arrangements that when the discharge is too great, the water surface in the cylinder *e* shall rise, and when too small that it shall descend. This is effected in the following manner:—A small closed cistern, *g*, is placed at the side of the portico of the entrance door of the building; this is supplied with water by the horizontal pipe, *r*, in communication with the vertical pipe, *h*, placed on the discharging main for the escape of air, which would otherwise collect within it, and greatly impede the discharge. In all cases of discharge of water through pipes, care must be taken that the air which may collect be readily let off.—*Vide* Buck's Account of the Montgomeryshire Canal Lock; Simms on Public Works of England, p. 8. The pipe, *h*, must be carried up the slope of the embankment, and communicate with the air above the level of the highest water in the reservoir. The cistern, *g*, is thus kept constantly supplied with water, and a communication is formed between it and the bottom of the cylinder, *e*, by the pipe, *k*. In the vertical part of this pipe are fixed two double-beat valves—described below—whose common spindle is fixed to the float, *n*; now if the surface of the water upon which *n* rests should rise beyond the proper level, then this float, *n*, also rises, and forcing up the spindle, closes the upper or discharge

valve from the cistern, *g*, and, as the valves are fixed on one spindle, of course simultaneously opens the lower one, so that the water which buoys up the float, *d*, in the cylinder, *e*, begins to flow out, and the consequent depression of *d* partially closes the flap-valve, *a*; and therefore the surface, *l*, *l*, begins to descend, and with it the float, *n*, which consequently opens the valve which had shut off the water from the cistern, *g*, and so preserves a nearly perfect equality between the supply and the consumption of the water.

In cases when the pressure upon a sluice is not great, the float, *n*, may be directly connected with the lever which works the sluice. Fig. 41 represents this simple apparatus: *a*, *a*, is the transverse section of the conduit, in which the sluice, *b*, moves vertically, and is connected by an adjustable link with an oscillating beam, *c*, jointed to the top of the short pillar, *d*. The other extremity of this beam is similarly connected to a hollow wrought-iron float, *e*, which is acted upon by the water whose surface is intended to be preserved at the same constant level, and the supply of which is derived from the conduit, *a*; if then the surface at *e* rise, the sluice is depressed, and the discharge by the conduit lessened, and *vice versâ*. This arrangement is evidently only suited to an open conduit, in which no great pressure can be brought upon the sluice; if applied to the mouth of a closed pipe with a great head of water pressing on it, the friction in the grooves of the sluice-frame would be so great as to require an enormous float, *e*, and the action could not fail to be of an irregular character.

The double-beat valve, invented by Hornblower (*vide* Pole on the Cornish Engine, pp. 85–88), is represented, fully opened, in the transverse section in Fig. 42, and isometrically in Fig. 43. It is intended that the water or steam should pass from A to B when the valve is opened, and that the communication between them be intercepted when it is shut. The shadings sloping to the left represent the movable parts; those to the right indicate the fixed parts of the valve. The value of its peculiar construction may be best appreciated by considering the tests of a good valve, which should, in the first place, evidently afford a large passage to the steam or water, with a small displacement; and, secondly, should be capable of being opened with a small force. These conditions are admirably fulfilled in the double-beat valve, which consists of a fixed part or seat C, formed by five partitions, which radiate from a central axis, and are joined below to a ring, *a*, and closed on top by a circular disc, in one piece with the partitions, and covering the spaces between them, and also

by a movable part, D, the valve proper, which is a sort of case surrounding the seat, C, and having a vertical motion sliding up and down the exterior edges of the partitions in C; this case is open on the top, and connected with its actuating rod, *n*, by the arms, *r*, *r*, when it is at the lowest point of its stroke, and shut, it bears upon the bevilled or conical surfaces, *a* and *a′*, which have but a very small breadth; when, on the contrary, it is raised, as in Fig. 42, it permits the passage of the water through the different openings shown by the bent arrows. It is evident that by this arrangement it is not necessary to raise the valve through any great height in order to afford a large passage to the water, thus satisfying the first test mentioned above; on the other hand, the valve, D, being pierced on its upper part by a circular opening nearly as great as that on the lower part, the force required to raise it is the excess of the pressure of the water or steam per square inch in A over that in B, multiplied into the difference of the circular areas above mentioned, this difference being evidently the annulus formed by the sum of the horizontal projections of the upper and lower conical surfaces, *a* and *a′*, shown in Fig. 42, projected down from the transverse section.

If this valve or case, D, had been a simple disc with bevilled edges, as in Fig. 44, we should have required to lift or start it a force equal to the excess of pressure in A over that in B, multiplied into the whole circular area of the top of the disc, *v*, *v*; and this would not only have to be provided by the prime mover, but a very greatly increased size and strength given to the rods, joints, &c. which actuate the valve. In a large disc-valve as, suppose, 12 inches diameter, the area being 113.1 sq. inches, and with an excess of pressure in A above that in B of 15 lbs. per square inch, it would require a force of 1696.5 lbs. to lift it. If in an equal double-beat valve each annulus was $\frac{3}{4}$ inch broad in the horizontal projection, the sum of their areas would be $(12^2 - 9^2) \times .7854 = 49.48$ square inches; thus the force required for the starting of such a double-beat valve is less than half that necessary for an equal disc valve, being $49.48 \times 15 = 742$ lbs., or 954 lbs. in favour of the double-beat valve, and so in proportion for pressures other than 15 lbs. In the particular case of the valves raised by the float, *n*, it may be, moreover, remarked, that the force necessary to raise them has to be applied but for a very short time, the instant it is raised, the pressure on each side is brought to a state nearly that of equilibrium; the less, then, the resistance to the float at the moment of raising the valve is, the more sensitive it becomes to any alteration in the surface of the water in *l*, *l*, with an absence of any irregular or jerking motion, the

flap valve, a, being consequently retained more steadily at its proper adjustment.

108. *Examples on Weirs.*—Let us, as in p. 72, exhibit the effect, on the quantity discharged per second, of the different values of m, namely, 0.60, 0.665, and between 0.662 and 0.595. —*Vide* § 77, pp. 56–57.

Thus, suppose an overfall of 1 ft. in width, having a depth of 1 ft. passing over: required the discharge in one second, the formula $\frac{2}{3} mlH\sqrt{2gH}$ then becomes $\frac{2}{3} \times m \times 8.024$, or $m \times 5.35$.

1. m = (§ 64, p. 43) 0.60, value of Q = 3.21 cb. ft.
2. m = (§ 64, p. 43) 0.665, " = 3.558
3. m = (§ 82, p. 61) 0.66654, " = 3.566
4. m = (§ 64, p. 44) 0.595, " = 3.183

In the following questions it is intended to show the effect of the function of the head or charge, $H\sqrt{H}$, which occurs in the formula for the discharge over weirs: a certain length is taken, and the discharge with a given head determined, and then this discharge being increased by a given quantity (xxv.), the corresponding increase of H is determined; in the same way, the discharge being doubled, it is sought (xxvi.) to determine the relative increase of the value of H.

(xxiv.) Calculate the discharge over a weir 1100 ft. long, the depth from the surface of still water to the crest of the weir being 0.75 ft., using .665 for the value of m, as given in the second case (§ 64, p. 43), we have $8.024 \times \frac{2}{3} \times .665 = 3.558$, and as $\sqrt{.75} = .886$. Hence $3.558 \times 1100 \times .75 \times .866 = 2544$ cb. ft. per sec. In Beardmore's Table II. we find the discharge for 1 ft. of length of weir with .75 ft. head, 138.88 cb. ft., and this being multiplied by 1100, gives 152768; but as all his Tables are calculated for the discharge per minute, dividing 152768 by 60, we obtain 2549.8, differing from that calculated above by 5.8; the coefficient used by Beardmore being .6665 (§ 82, p. 61), giving 3.566, instead of 3.558 used above.

(xxv.) To what height upon the crest would the water rise if the discharge was increased to 3000 cb. ft. per sec.? We have from these data,

$$3.558 \times 1100 \sqrt{H^3} = 3000.$$

Hence

$$H = \sqrt[3]{\left(\frac{3000}{3.558 \times 1100}\right)^2} = \sqrt[3]{(.766)^2} = \sqrt[3]{.587} = .837,$$

being an increase of $.837 - .75 = .087$ ft., the increase of H being only 11.6 per cent., and that of Q being 17.9 per cent.

The least laborious method of finding cube roots, when no table of logarithms is at hand, is the following:—Assume a number whose cube is nearly equal to the given number, then as twice this cube, plus the given number, is to twice the given number, plus the assumed cube, so is the assumed root to the true; in this case, for $\sqrt[3]{.587}$ first assume .8, which gives .512; secondly, assume .84, which gives .593. Hence

$$\overline{2 \times .593} + .587 : \overline{2 \times .587} + .593 :: .84 : .837.$$

And by logarithms we have log of $.587 = \bar{1}.7686381$, which, divided by 3, gives $3)\,\underline{\bar{3}.7686381}$

$\bar{1}.9228793$ answering to .83728.

(XXVI.) If the discharge in (XXIV.) had been doubled, calculate the depth of water flowing over the crest. The discharge in (XXIV.) being doubled, gives ($2 \times 2544 =$) 5088 cb. ft. per second on a length of 1100 ft. Hence

$$H = \sqrt[3]{\left(\frac{5088}{3.588 \times 1100}\right)^2} = \sqrt[3]{1.69}, \text{ and}$$

log. of $1.69 = 0.2278867$, which, divided by 3, gives 0.0759622, answering to 1.19131; deducting .75, we have .44 ft. for the rise, to be added to the first supposed .75, in order to obtain a double discharge, so that, instead of 1.50, i. e. twice the original head, we have but 1.19 ft. on the crest of the weir for twice the original discharge; it is, in fact, evident that .75 is multiplied by $\sqrt[3]{2^2} = 1.5866$, instead of by 2.

If the length of the weir in (XXIV.) had been reduced one-half, namely, to 550 ft., calculate the head to which the water would rise upon the crest, the discharge being the same, namely, 2544 cb. ft. per sec. We have now

$$Q = 2544 = 3.558 \times 550 \times H^{\frac{3}{2}}.$$

Hence

$$H = \left(\frac{2544}{1956.9}\right)^{\frac{2}{3}} = 1.191 \text{ ft.}$$

(XXVII.) The construction of the weir at Killaloe ("Selection of Specifications") has the peculiarity of not being level on part of the crest. The inclination being 1 in 214, and the rise 1.5 ft., the length with that slope must be $1.5 \times 214 = 321$ ft., we have, therefore, as the weir is 1100 ft. long, 779 ft. for the level portion, and 321 ft. at an inclination of 1 in 214. Calculate the total quantity discharged over this weir when the

depth of water on the level part is 1.8 ft., so as to have 0.3 ft. on the highest part of the crest at the west abutment. If then we divide this sloping part into eight lengths, of 40 ft. each, and calculate the discharge over each length with a head equal to the arithmetic mean of the head at each extremity of the 40 ft. lengths, the discharges will be sufficiently near the truth. The increase of depth on each 40 ft. is evidently $\frac{40}{214}$ ft., equal to 0.18691 ft., and as the depth over the highest point at the west abutment is, by the terms of the question 0.3 ft., the mean depth for the first 40 ft. is

$$\frac{0.3 + \overline{0.3 + .18691}}{2} = 0.393455 \text{ ft.};$$

to obtain the second, third, &c. we have but to add to this successively 0.18691, and consequently obtain the following numbers:—.393, .580, .767, .954, 1.141, 1.328, 1.702, 1.795; which, being multiplied by their respective square roots, give .2468, .442, .672, .932, 1.219, 1.530, 1.864, 2.220.

Hence the eight several discharges through the 40 ft. lengths are found by multiplying the common part of the formula (§ 55) $\frac{2}{3} m, l, H\sqrt{H}.\sqrt{2g}$, that is,. $3.558 \times 40 = 142.3$, into the values of $H\sqrt{H}$ given above, and, adding these, we have the total discharge over the sloping part of this weir 1299 cb. ft. per sec. And for the length of 780 ft. of level crest with 1.8 ft. head, we have 6700 cb. ft. per second. Hence the total discharge is 7999 cb. ft. per second. As $8 \times 40 = 320$ ft., and the length of sloping portion is 321 ft., we must add one foot to 779, the length of the level portion.

(XXVIII.) In the weirs on the Shannon constructed by the Commissioners, it was requisite that salmon-gaps should be constructed, so that the fish be able to migrate up stream at the weirs during such periods as might not afford sufficient depth of water if the whole quantity were uniformly distributed over the total length of the weir. These were 10 ft. wide, and the crest 1.5 ft. below that of the weir. Calculate the quantity flowing down three of these salmon-gaps, the water on the level part of the crest being 0.6 ft. deep. Here

$$H = 1.5 + 0.6 = 2.1, \text{ and}$$

$$3Q = 3 \times 3.558 \times 10 \times 2.1\sqrt{2.1} = 324.8 \text{ cb. ft.}$$

(XXIX.) A feeder or water-course along the side of a valley is required to be augmented by the streams and springs above

its level. It is required to determine their total volume. For this purpose the several courses are dammed up at convenient and suitable places, and a narrow board provided, in which is cut an opening for the overfall 1 ft. long and 0.5 feet deep; it being reasonably surmised that this would be sufficient to gauge the largest of the streams; and another piece was prepared that, when attached to the former, would reduce the length to 0.5 ft. for the smaller. Calculate the total quantity delivered by the five following streams and springs:

No. 1, on being dammed up, flowed over the 1 ft. opening .37 feet deep. Hence $Q = 3.558 \times .37\sqrt{37} = .8$ cb. ft.

No. 2, at .5 ft. in length of overfall, rose to .41 ft. in depth Hence $Q = 3.558 \times .5 \times .41\sqrt{.41} = .467$ cb. ft. per sec.

No. 3, at 1 ft. length, was .29 ft. high on the overfall, and

$$Q = 3.558 \times .29\sqrt{.29} = .555 \text{ cb. ft. per sec.}$$

No. 4, at .5 in length, rose .19 ft., $3.21 \times .19\sqrt{.19} = .133$ cb. ft. per sec.

No. 5, being a small spring, was not measured by the overfall; but being banked up, a pipe, .0416 ft. in diameter, was let through the dam, and when the surface had become stationary, and consequently the discharge through the pipe equal to the supply from the spring, it was gauged into a vessel marked for 1 and 2, &c. imperial gallons; the time required to reach the former was 32 seconds. Hence the spring gave 0.005 cb. ft. per sec., as 6.25 gallons make one cubic foot.

The total quantity, therefore, received by the aqueduct from the lateral springs and streams above its level amounted to 1.96 cb. ft. per second.

(xxx.) On the Manchester water-works weirs are constructed across some of the lateral mountain streams which supply the reservoirs, so that the higher velocity which the water has when flowing over at the greater depths may keep separate the turbid water, unfit for the town supply, from the clear. Fig. 45 gives a diagram transverse section of this weir. In heavy or sudden rains these streams bring down very rapidly water discoloured by peat and earth, and unfit for domestic use; but in fine weather the quantity is much reduced, and the water clear and suitable for the mains of the town. Fig. 45 represents a transverse section of the water-course which is carried through the masonry of the weir, conveying clear water from other streams, across the valley in which the weir is placed, and so serving as an aqueduct; at the top

this is open, as at $m\ n$, and when the water flows over at a small depth, that is, when it is clear, it falls into the channel, and is conveyed by it eventually into the main which supplies the town; but if it rise and discharge a greater body of water, the increased velocity projects it beyond the edge of the opening, and it thus passes over the longitudinal opening, and flows down to the compensation reservoir for the supply of power to the mills situated on the river. By referring to §§ 30, 31 we find the means of calculating the curve of any issuing jet of water. But in this case we have a different velocity, and therefore a different parabola for every lamina into which we may suppose the water divided. Fig. 46 represents the different paths taken by each, that for the mean velocity at $\frac{4}{9}$ths of the depth being drawn in a full line; hence those above will tend to depress the curve, and those below, on the contrary, to carry it more up towards the horizontal line; we may therefore suppose the whole sheet of water to be carried out in a curve at top and bottom parallel to that of the mean velocity. If therefore we put H_1 for the depth of the water flowing over the weir, the mean velocity being $\frac{2}{3}$rds of that at the bottom, we have $v = \frac{2}{3} \times 8.024 \times \sqrt{H_1}$ for this mean velocity, and the curve taken by the lowest lamina is that due to a head $\frac{4}{9} H_1$, for in the expression (§ 48, p. 33)

$$z' = \frac{4}{9}\left(\frac{H\sqrt{H} - h\sqrt{h}}{H - h}\right)^2,$$

if we put $h = 0$ the resulting value of z' is $\frac{4}{9} H$. Now in Fig. 45 we have $x = 1$ ft., $y = 0.83$ ft.; hence from § 31,

$$\frac{4}{9} H_1 = \frac{y^2}{4x} = \frac{0.83^2}{4} = 0.1722 \therefore H_1 = 9 \times .1722 \div 4 = 0.3874 \text{ ft.}$$

So then when the water flowing over has a depth at or greater than 0.3874 ft. it is carried completely over the longitudinal opening; it is then necessary to gauge the stream in wet seasons, and so proportion x to y that the volume of water, from the head necessary to discharge it, have velocity sufficient to pass over the opening $m\ n$; at lesser depths it strikes against the point, and in part enters the clear water-channel, and in part flows over the weir; for this reason it is necessary to have a cover of timber, that the attendant may turn down upon the opening during such period if, at the commencement or end of a flood, the water should be turbid at such a depth as would not completely pass over the opening $m\ n$. Calculate at what

depth the water all flows in. If we suppose in Fig. 47 that $mr = H_1$, which we may do, though it be not normal to the axis of the sheet of water, then $y + H_1 = 0.83$ and $H_1 = 0.83 - y$, also $\frac{4}{9} H_1 = \frac{y^2}{4}$ in this substitute for H_1 its value above, we have

$$\frac{4}{9}(0.83 - y) = \frac{y^2}{4} \text{ and } \frac{4}{9} \times 0.83 = \frac{y^2}{4} + \frac{4}{9}y;$$

$$\text{or } \sqrt{2.266} = y + \frac{8}{9}, \text{ or } 1.5 - .888' = y;$$

$$\text{hence } y = .612 \text{ and } H_1 = .8333 - .6112 = .2221 \text{ ft.}$$

If, then, we observe in ordinary seasons a stream discharging 26.6 cb. feet per sec., the water being clear, and the most convenient length of the crest of the overfall being 60 ft.,—we may, having selected some convenient depth $n_1 n$, so adjust the opening mn that the whole of the clear water may fall into it. As a first step, we must calculate H_1, for the length 60 ft., and discharge 26.6, if m be taken equal to .6665 (§ 82), we have, therefore, $H_1 = \sqrt[3]{\left(\frac{26.6}{3.556 \times 60}\right)^2} = .25$ (§ 71), and the vertical depth of n (Fig. 47) below the crest at m being given, we may calculate, first, the value of y, that is $n_1 n$, so that the curve of the upper surface of the sheet of water flowing over the crest may fall within the point n, and the whole stream be carried down the clear water aqueduct. Let mn_1 be taken at 1 ft., then from

$$y^2 = \frac{4v^2 x}{2g} \text{ (§ 30), we have } y = \frac{2v\sqrt{x}}{\sqrt{2g}}, \text{ and}$$

substituting for v its value in this particular case where $H_1 = .25$, we have $v = \frac{2}{3}\sqrt{2gH_1} = \frac{2}{3} \times 8.024\sqrt{.25}$ (§ 48), and also for x its value, which is $mn_1 + H_1 = 1.25$ ft. Hence

$$y = \frac{2 \times \frac{2}{3} \times 8.024\sqrt{.3125}}{8.024} = \frac{4}{3} \times .559 = .745 \text{ ft.}$$

And secondly, we may calculate at what amount of discharge and head H_1 the curve of the lower parabola of the sheet of water will pass completely over the opening mn, and so the stream, now turbid, be all carried over the clear water aqueduct into the settling and compensation reservoirs. Call the sought depth D, and as x is now 1 ft., we have $y = .745 = \frac{2 \times \frac{2}{3} \times 8.024\sqrt{D}}{8.024}$

$= \frac{4}{3}\sqrt{D}$ and $D = \left(\frac{3}{4} \cdot .745\right)^2 = .31248$, the discharge being about 37 cb. ft. per sec.

(xxxi.) To determine generally the relation between the length and depth of weir having the same discharge, put

$$\frac{2}{3} \times m \times l_1 \times h_1 \sqrt{h_1} = \frac{2}{3} m \times l_2 \times h_2 \sqrt{h_2};$$

hence $$h_1^{\frac{3}{2}} : h_2^{\frac{3}{2}} : : l_2 : l_1,$$

and $$h_1 : h_2 : : l_2^{\frac{2}{3}} : l_1^{\frac{2}{3}},$$

$$\therefore h_1 = h_2 \left(\frac{l_2}{l_1}\right)^{\frac{2}{3}},$$

$$\log h_1 = \log h_2 + \frac{2}{3}(\log l_2 - \log l_1).$$

Calculate the height to which the water upon a weir 545 ft. long will rise when it is flowing down from another weir higher up upon the same river, whose length is 750 ft., and on which it rises 0.68 ft., it being supposed that no additional supply has been received in the intervening part of the course.

Here
$$\begin{array}{rl} \log l_2 = & 2.8750613 \\ \log l_1 = & 2.7363965 \\ \hline & 0.1386648 \times \frac{2}{3} = 0.0924432 \\ \text{and } \log h_2 = & 1.8325089 \\ & .0924432 \\ \hline & 1.9249521 \text{ and } h_1 = 0.84 \text{ ft.} \end{array}$$

(xxxii.) In the construction of reservoirs it is necessary to have a weir whose crest is on the level of the intended top-water line, with reference to which line the height of the embankment and of the puddle-wall must be designed. The length of this weir must be such that the water of a maximum rain-fall shall not rise above a certain height. We may take the greatest rain-fall at 2 inches in 24 hours: this depth must be multiplied into the area of district which drains into the reservoir. We thus have, first, the total volume of water; and secondly, supposing the rain to have fallen at a uniform rate during the 24 hours, or at least to have been delivered by the water-courses into the reservoir at a uniform rate, we thence obtain the quantity per minute or per second which this weir must discharge. We then assign a certain depth

upon the crest, to which the water must be limited, and consequently from the depth H and discharge Q we may obtain L. Thus, suppose the area of the rain-basin or district draining into the reservoir were 6536 acres, and the maximum depth of rain in 24 hours to be 2 inches, we reduce both to the same unit of feet. The acre contains 10 square chains of 66 feet each, $66^2 \times 10 = 43560$ sq. ft., and 2 inches $= .1666'$ ft.: hence, $43560 \times 6536 \times 0.1666' = 47451360$ cb. ft. in 24 hours, which, reducing to seconds, we have $24 \times 60 \times 60 = 86400$; and dividing $47451360 \div 86400 = 549.2$ cb. ft. per second, entering the reservoir the length of weir to discharge this with a rise on the crest of 1.5 ft. is found

$$\frac{549.2}{\frac{2}{3} \times .66 \times 1.5 \sqrt{1.5} \times 8.024} = \frac{549.2}{6.48} = 84.75 \text{ ft.}$$

As, however, the sluices for discharging the storage would be opened, the rise upon the crest could be readily kept down to 1 ft.

EXAMPLES ON CHAPTER II.

FLOW OF WATER UNDER A VARIABLE HEAD.

109. In § 88 we have the formula—

$$T = \frac{2 A \sqrt{H}}{mS \sqrt{2g}}.$$

(XXXIII.) This has been made use of by Dr. Young to determine the value of m. A tube 1 inch in diameter is filled for 9 inches with mercury; at the bottom is an orifice $\frac{1}{20}$ inch in diameter; the observed time of its total discharge was 140 seconds. Solving the above equation for m, we have—

$$m = \frac{2A \sqrt{H}}{T . S . \sqrt{2g}}.$$

Changing the measures from inches into feet, we have—

$$2A = .083^2 \times .7854 \times 2 = 0.0109 \text{ sq. ft. and } \sqrt{.75} = 0.866 \text{ ft.}$$

$$S = \frac{1}{400} \times .00545 = .0000136 \text{ sq. ft.}$$

So that—

$$m = \frac{.0109 \times 0.866}{140 \times .0000136 \times 8.024} = \frac{.0094394}{.0152777} = 0.62.$$

However, on account of the circular or vortex motion of the fluid at very small depths, no formulæ which give the time for *complete* exhaustion are quite exact. Mercury is, probably, less affected by this motion than water, with which a funnel-shaped vortex is formed over the orifice, this drawing in the air renders the discharge irregular, and reduces the orifice, so that the formula—

$$t = \frac{2A\,(\sqrt{H} - \sqrt{h})}{mS\,\sqrt{2g}},$$

in § 89, may be considered to give more exact results. The same author has used it also, as in the following experiment.

(XXXIV.) A prismatic vessel, having a diameter of 5.747 inches, has an orifice 0.2 inch at the bottom, and its surface is observed to sink from 16 inches to 1 foot of depth in 53 seconds. Transposing as before, we have—

$$m = \frac{2A\,(\sqrt{H} - \sqrt{h})}{tS\,\sqrt{2g}},$$

H being 1.33′ ft., and h = 1 ft., the value of $(\sqrt{1.33} - \sqrt{1})$ is 1.16 − 1 = 0.16 ft. The diameter of the vessel being 5.747 inches, or 0.4783 ft., the value of A will be $.4783^2 \times .7854$ = .1797 sq. ft.; also $S = .0166^2 \times .7854 = .000218$ sq. ft. Hence—

$$m = \frac{2 \times .1797 \times (1.16 - 1 =)\ 0.16}{53 \times .000218 \times 8.024} = \frac{.0575}{.0930} = .62.$$

(XXXV.) A prismatic basin, whose horizontal section is a square of 3 ft. in the side, has at the bottom an orifice 0.09 ft. in diameter; it is filled up to a depth of 6 ft. above the centre of the orifice. Calculate the *time* required for the surface to descend 3.5 ft., counting from the moment of opening the orifice. Here $A = 3 \times 3 = 9$ sq. ft., $S = 0.09^2 \times .7854 = .00636$ sq. ft.; $H = 6$, and $h = 6 - 3.5 = 2.5$, m being 0.61; therefore from the formula—

$$t = \frac{2 \times 9\ (2.449 - 1.581)}{0.61 \times 0.00636 \times 8.24} = \frac{15.624}{.03113} = 502'' = 8'\ 22''.$$

(XXXVI.) With the same dimensions calculate the time required for the surface to descend 2 ft. Here $h = 6 - 2 = 4$, and $\sqrt{H} - \sqrt{h} = 0.449$ ft.; therefore—

$$t = \frac{18 \times .449}{.03113} = 259.6 = 4'\ 20''$$

(XXXVII.) Again, suppose the descent of the surface to be 5 ft., calculate the time, $h = 6 - 5 = 1$, and $\sqrt{H} - \sqrt{h} = 1.449$, so that—

$$t = \frac{18 \times 1.449}{.03113} = \frac{26.082}{.03113} = 837''.84 = 13'58''.$$

(XXXVIII.) § 91. *Mean hydraulic charge.* –Let us suppose in any prismatic vessel receiving no supply, that the head, at the instant of opening the orifice of discharge, was 6 ft. = H, and at closing it had decreased to 5 ft. = h, calculate the mean constant charge at which, in the same time, the orifice would discharge the same volume of water; the vessel being now, necessarily, supposed to receive that same constant quantity which it discharges with a uniform velocity.

The formula is—

$$H' = \left(\frac{H - h}{2(\sqrt{H} - \sqrt{h})}\right)^2 = \left(\frac{6 - 5}{2\,(2.449 - 2.236)}\right)^2 = \left(\frac{1}{.426}\right)^2 = 5.508 \text{ ft.}$$

If h be taken equal to 4, then $H' = 4.96$; if equal to 3, $H' = 4.376$; if $h = 2$, then $H' = 3.732$; and when $h = 0$, we have $H' = 1.5$.

If in 10″ we observe the surface to fall 2 ft., determine the coefficient of discharge.

If $A = 6$ ft., $S = .01$, and $T = 10''$, then H being $= 6$, and $h = 4$, we have $Q' = 12$ cb. ft., and $Q = 1.2$, and $\sqrt{H'} = 2.227$.

Hence—

$$m = \frac{1.2}{0.1 \times 8.024 \times 2.227} = \frac{1.2}{1.787} = 0.67.$$

(XXXIX.) § 92, p. 67. A reservoir, half an acre in area, with sides nearly vertical, so that it may be considered prismatic, receiving a stream which yields 9 cb. ft. per second, discharges through a sluice 4 ft. wide, which is raised 2 ft.; calculate the time required to lower the surface 5 ft., the charge upon the centre of the sluice, when opened, being 10 ft. From the formula given at the end of § 92, we have, substituting the numerical values, $A = 21780$ sq. ft. the acre, being 43560 sq. ft.; $S = 8$ sq. ft, m being found 0.70, and $h = 10 - 5 = 5$, also $q = 9$ cb. ft. per second.

$$t = \frac{2 \times 21780}{(.7 \times 8 \times 8.024)^2} \left\{ .7 \times 8 \times 8.024 \left(\sqrt{10} - \sqrt{5}\right) + 2.303 \times 9 \times \log \frac{.7 \times 8 \times 8.024\sqrt{10} - 9}{.7 \times 8 \times 8.024\sqrt{5} - 9} \right\}$$

In this we have $.7 \times 8 \times 8.024 = 44.9$, and $\sqrt{10} - \sqrt{5} = 3.162 - 2.236 = .926$.

Hence—

$$t = \frac{43560}{2016}\left\{44.9 \times .926 + 20.7 \log 1.455\right\}$$
$$= 21.607\{41.6 + 3.37\} = 972'' = 16', 12''.$$

If q, the constant supply received by the reservoir, had been 20 cb. ft. per second, then—

$$\frac{(44.9 \times 3.162) - 20}{(44.9 \times 2.236) - 20} = \frac{121.97}{80.40} = 1.517,$$

the log of which is 0.1809856 (in the former case subtracting 9 we had $\frac{132.97}{90.4} = 1.455$, the log being 0.1628630), and the value of t is now $21.607\ \{41.6 + 2.303 \times 20 \times .181\} = 1759 = 29'\ 19''$ to lower the surface 5 ft.

(XL.) Referring to the latter part of § 92, in order to determine the depth which the surface would descend in a given interval of time, the formula must be arranged so as to separate the factors of $\sqrt{H}$ from $\sqrt{h}$, then transposing, so as to make the left-hand side = 0, we have

$$t - \frac{2A}{(mS\sqrt{2g})^2}\{mS\sqrt{2g}\sqrt{H} + 2.303 \times q \times \log(mS\sqrt{2g}\sqrt{H} - q)\}$$
$$+ \frac{2A}{(mS\sqrt{2g})^2}\{mS\sqrt{2g}\sqrt{h} + 2.303 \times q \times \log.(mS\sqrt{2g}\sqrt{h} - q\} = 0.$$

Let us suppose all the letters to have their former values, t being taken at 20 minutes, calculate the value of h—

$$(t =)\ 1200'' - \frac{43560}{2016}\ \{44.9 \times 3.162 + 20.73 \times \log 133\} =$$
$$1200 - 4020 = -2820,$$

and thus we have—

$$21.61 \times \{44.9\sqrt{h} + 20.73 \times \log(44.9\sqrt{h} - 9)\} - 2820 = 0,$$

when the true value of h is substituted. To further prepare this last expression for the tentative determination of h, we must multiply out by 21.61, hence—

$$970.3\sqrt{h} + 448 \log(44.9\sqrt{h} - 9) - 2820 = 0.$$

If we take at first—

$\sqrt{h} = 2$, the equation becomes $- \quad 25 = 0$,
$\sqrt{h} = 2.4 \quad$,, $\quad$,, $\quad + \ 422 = 0$,
$\sqrt{h} = 2.01 \quad$,, $\quad$,, $\quad - \quad 14 = 0$,
$\sqrt{h} = 2.1 \quad$,, $\quad$,, $\quad + \ 82.7 = 0$,
$\sqrt{h} = 2.03 \quad$,, $\quad$,, $\quad + \ 7.44 = 0$,
$\sqrt{h} = 2.023 \quad$,, $\quad$,, $\quad - \quad 0.1 = 0$, and $h = 4.09$.

The surface, therefore, descends 5.9 feet in 20′.

(XLI.) § 93. A pond, whose area is 12000 square feet, has an overfall outlet 3 feet wide, and at the commencement of the discharge has a head of 2.8 feet, calculate the length of time required for the surface to descend 1 foot, it being supposed that no supply is received.

We have then $H = 2.8$, and $h = 2.8 - 1 = 1.8$, the value of m being taken at 0.61.

The formula—

$$t = \frac{3A}{ml\sqrt{2g}}\left(\frac{1}{\sqrt{h}} - \frac{1}{\sqrt{H}}\right)$$

being put into numbers for this question, we have—

$$t = \frac{3 \times 12000}{.61 \times 3 \times 8.024}\left(\frac{1}{\sqrt{1.8}} - \frac{1}{\sqrt{2.8}}\right) = 2452\left(\frac{1}{1.34} - \frac{1}{1.673}\right)$$

$$= \frac{2452}{1.34} - \frac{2452}{1.673} = 1830 - 1466 = 364'' = 6'\ 4''.$$

Calculate the time in which the surface descends 0.5 feet. In this case $h = 2.8 - .5 = 2.3$, and $\frac{1}{\sqrt{2.3}} = \frac{1}{1.516}$. Hence—

$$\frac{2452}{1.516} - \frac{2452}{1.673} = 1617 - 1466 = 151'' = 2'\ 31''.$$

Again, if we suppose the depth descended to be 1.5, and all the other quantities remain the same, we shall thus have

$h = 2.8 - 1.5 = 1.3$, and $\frac{1}{\sqrt{1.3}} = \frac{1}{1.14}$, so that

$$t = \frac{2452}{1.14} - \frac{2452}{1.673} = 2151 - 1466 = 685'' = 11'\ 25'';$$

the depths then being 0.5, 1, 1.5 feet, the corresponding intervals are 2′ 31″, 6′ 4″, 11′ 5″. If $h = 0$, it is evident that t becomes infinite, as $\frac{2452}{0}$ = infinity, and so also of any finite number in the numerator, arising from any other data. If the depth sunk had been nearly equal to the whole charge at the

commencement, as, suppose, 2.4, so that $h = 2.8 - 2.4 = 0.4$, then $\frac{1}{\sqrt{0.4}} = \frac{1}{.6324}$, and

$$t = \frac{2452}{.6324} - \frac{2452}{1.673} = 3877 - 1466 = 2411'' = 40'\ 11''.$$

(XLII.) § 95. In question XIII., page 77, Fig. 31, taken from D'Aubuisson, the time of filling the lower part of a canal lock, on the Canal du Midi, is calculated, i. e. up to the level of the centre of the sluices, placed in the upper pair of gates; we can now, by the 2nd case of § 95, calculate the time of filling up to the level of the upper reach, from the centre of the sluice doors, which, added to the 25″ as determined in XIII., will give the total time. Substituting in the formula—

$$T = \frac{2A}{mS\sqrt{2g}} \times \sqrt{H},$$

the several numerical values given at page 77, we shall have—

$$T = \frac{2 \times 3503.6}{.548 \times 13.532 \times 8.024} \times \sqrt{6.3945},$$

that is—

$$\frac{7007.2}{59.5} \times 2.53 = 298 = 4'\ 58'',$$

to which adding 25″, we have 5′ 23″ as the total time of filling a lock of such dimensions.

(XLIII.) The locks on the Montgomeryshire canal have a length of 81 and width of 7.75 feet; and at one, named the Upper Belun Lock, the lift or rise was 7 feet. A pipe leads the water from the upper level, and discharges below the surface of the lower level in the lock chamber, the diameter of which is 2 feet. As the mouth of this pipe is a square, 2 feet in the side, gradually altered into a circular pipe, 2 feet in diameter, we may take $m = 1$, a result which is justified by comparing the observed time of filling this lock with that calculated by the formula—

$$T = \frac{2A}{mS\sqrt{2g}}\sqrt{H},$$

when m is put equal to unity, for

$$\frac{2 \times 81 \times 7.75}{1 \times 2^2 \times \cdot 7854 \times 8.024} \times 2.645 = 132'' = 2'\ 12''$$

the observed time being 2′ 10″.

CHAPTER III.

FLOW OF WATER THROUGH PIPES, ARTIFICIAL CHANNELS, AND RIVERS.

110. Gravity is the sole force that acts upon a mass of water left to itself in a bed of any form; it produces all the motion which takes place,—the inclination of the surface of the water in the channel is the immediate cause of motion, being that which enables gravity to act: and thus the measure of this force is in ft. per second, $g \times \sin i$. If, then, water flowing in a channel or pipe, and subject to this constant accelerating force, meet with no resistance, it will descend with an increasing velocity which would never be found uniform. But observation and experience show that in open channels and pipes, even those of very great slope, the flow very soon becomes uniform. Bossut made the following experiment to prove this truth directly:—Having constructed a canal in wood, 650 ft. long, with a slope of 1 in 10, and marked off equal spaces of 108 ft. each, it was found that the water traversed each space, except the first, in equal times. There must then exist a retarding force, which destroys at each instant the effect of the accelerating force, and which is necessarily equal to it.

But in pipes, channels, &c., there can be no retarding force but that which arises from the resistance of the sides or bed: and of its existence we cannot doubt, for the simple experiment of observing the discharge through a tube in a certain time, and again when the tube has been lengthened—all else remaining the same—proves that the time required to give a certain volume of water has been increased also; and this can only arise from the fact that the tube, or other channel, by reason of its increased length, offered a greater resistance to the velocity. The surface thus opposed motion.

To these retarding forces the name of Friction has been applied: though, from the difference between the laws of friction of water flowing over its resisting bed, and the friction of solid bodies sliding upon each other, we must look upon it as the application of an old word in a new sense, in preference to adding a new term to express this peculiar resistance. It may be useful to state here briefly the laws of friction of solid bodies, with the view of showing this contrariety.

111. *First Law.*—Experiment has shown that the friction or resistance to motion of bodies, sliding upon their surfaces of contact, is directly proportional to the force or weight pressing the two surfaces together, and differs only with the nature of the sliding surfaces, as wood, brass, iron, &c.

Second Law.—The amount of friction is independent of the extent of the surface pressed, provided the whole amount of the pressure remains the same, and that the substance of the surface pressed is the same.

Third Law.—The friction of a body, when in a state of continuous motion, bears a constant ratio to the pressure upon it, which is the same, whatever may be the velocity of the motion,—it is, in other words, independent of the velocity. Thus the first only of these laws can be expressed algebraically.

112. In the case of fluids, it has been shown that the resistance to motion which we observe, and which has been called friction also, is, on the contrary—

First Law.—Independent of the pressure, that is, that the resistance to motion in a pipe with a head of, suppose, 100 ft., is the same as if the head were but 50 ft., or any other height. Dubuat had proved this by experiments on the oscillation of water in syphons, which the author of the article "River" (Encyclopædia Britannica) has thus improved upon:—

Two vessels ABCD, *abcd* (Fig. 48), were connected by the syphon EFG *gfe*, which turned round in the short tubes E and *e*, without allowing any water to escape; the axis of these tubes being in one right line. The vessels were about ten inches deep, and the branches FG, *fg* of the syphon were about five feet long. The vessels were set on two tables of equal height, and (the hole *e* being stopped) the vessel ABCD, and the whole syphon, were filled with water, and the water was poured into the vessel *abcd* till it stood at a certain height LM. The syphon was then turned into a horizontal position, and the plug drawn out of *e*, and the time carefully noted which the water employed in rising to the level HK*kh* in both vessels. The whole apparatus was now inclined so that the water ran back into ABCD. The syphon was now put in a vertical position, and the experiment repeated: no sensible or regular difference was observed in the time; yet in this experiment the pressure on the part G*g* of the syphon was more than six times greater than before. As it was thought that the friction on this small part (only six inches) was too small a portion of the whole obstruction, various additional obstructions were put into this part of the syphon, and it was even lengthened to nine feet;

but still no remarkable difference was observed. It was even thought that the times were less when the syphon was vertical; nor has any variation ever been observed in the friction of water upon glass, lead, iron, wood, &c. (Principes d'Hydraulique, Tome i., §§ 34 and 36, Dubuat.)

Second Law.—The resistance is, at any one velocity, proportional to the surface exposed to the action of the flowing water. In order to obtain an expression for this law, we may remark, first, that in any channel or pipe the resistance of the surface (Fig. 49) ABCD *abcd* is, from the mutual adhesion of the lamina into which we may suppose the water divided, and which couches or lamina are indicated by the dotted lines in the figure, shared by all the particles in the volume of water between the two transverse planes whose distance is Aa,—that nearest the sides being most retarded, and each in succession less and less influenced. The greater, then, that surface is, the greater is the resistance. And, again, the greater the volume upon which this retarding action of the surface acts, the less will the velocity of the first and therefore of each successive lamina be reduced: and thus we have the resistance directly proportional to the surface, and inversely as the volume; or as the surface is $Aa \times (ab + bc + cd)$, and the volume $Aa \times$ (area *abcd*), we may evidently in $\dfrac{Aa \times (ab + bc + cd)}{Aa \times (\text{area } abcd)}$ strike out from each the length Aa of the channel common to both. The length $ab + bc + cd$ is called the border, or the wetted perimeter: and thus we have the resistance directly proportional to the border, and inversely as the area of the transverse section perpendicular to the axis of the stream. If, then, we put S for the area of the section, and C for the contour of the border, we have the resistance proportional to $\dfrac{C}{S}$.

From this expression it is evident that many geometrical questions arise in designing the best form of channel, rectangular or trapezoidal, to convey given quantities of water: the same area having, with the same condition as to ratio of slopes, a great number of different borders, and one a minimum, and, *vice versâ*, the same border having a number of different sectional areas, and one a maximum.

Third Law.—The resistance is proportional to the square of the velocity nearly, the border being constant. For the number of particles drawn in one second from their adhesion to the sides of the channel or pipe is proportional to the number of ft. per second with which the water is moving, that is, to the velocity, and the force with which they are drawn is also as the same

number of ft. per second, or the same velocity: and thus the passive resistance of the wetted border to the flow of the water is proportional to the product of the velocity into the velocity; this part, then, of the expression for the resistance is represented by av^2, a being a constant, determined hereafter. Experimenters have shown that this gives the resistance a very little too high, and that with velocities increased in the ratio 2, 3, 4, &c., it is not represented by $a \times 4$, $a \times 9$, $a \times 16$, &c., but more nearly by adding the simple power of the velocity, thus, $a\,(v^2 + bv)$, the series of numbers $v^2 + v$ not increasing so fast as v^2.

Fourth Law.—In gases and elastic fluids we also have the friction proportional to the specific gravity or density.

In order to obtain from these laws a formula for the discharge of water through pipes and channels, we must make use of the well-known principle, that when any body is moving with a uniform velocity, the accelerating are necessarily equal to the retarding forces: for if the accelerating forces be supposed greater than the retarding, the velocity must increase; and if they should become less, then the velocity must, on the other hand, decrease. Our object must now be, as in the former chapters, to find, first the mean velocity, for this multiplied into the area gives the discharge with a given inclination: and we can thus solve practical questions, such as the requisite dimensions of pipe or channel to convey a given quantity of water, &c., &c. Now in any pipe or channel, whose length is l, and whose height, from the surface of the supply to the point of discharge or extremity of l, is represented by h, we have the accelerating force expressed by $\frac{h}{l} \times g$, or sin inclination of surface into gravity.

The retarding forces are, from the laws above given, neglecting bv, proportional to

$$(a) \quad \ldots\ldots\ldots \quad \frac{C}{S} \times v^2,$$

and therefore we have

$$(b) \quad \ldots\ldots \quad \frac{h}{l} \times g = a \times \frac{C}{S} \times v^2;$$

a being some constant quantity to be determined from experiment. If the formula be correct, all good experiments will give the same value for a, that quantity by which the right-hand side of (b) must be multiplied to produce the equality. We may, however, simplify the expression by dividing out by g, and thus we have

$$(c) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \frac{h}{l} = \frac{a}{g} \times \frac{C}{S} \times v^2,$$

and as g is constant, put $\frac{a}{g} = a'$, which must be constant if a be so; solving, then, for a', we have

$$(d) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \frac{h}{l} \times \frac{S}{C} \times \frac{1}{v^2} = a'.$$

Substituting the actual data of experiments for the letters of the left-hand side, we obtain the value of a', and comparing different experiments together, we find that it remains very nearly the same in all.

The celebrated Smeaton has given in his Reports (vol. II. p. 297) a series of experiments on the velocity of water flowing through pipes under pressure. One of these, No. 7, had the following data:—Diameter of pipe, $4\frac{1}{2}$ inches, or 0.375 ft.; length, 14637 ft.; fall or head, 51.5 ft.; and $v = 1.815$ ft. Hence $\frac{51.5}{14637} \times \frac{0.375^2 \times .7854}{0.375 \times 3.1415} \times \frac{1}{1.815^2} = .0001 = a'$.

The quantity discharged is given by Smeaton in Scotch pints, which he states contain 103.4 cb. inches, and therefore the number of cb. ft. in one pint is $\frac{103.4}{1728} = 0.05984$, and as 200 pints per minute were discharged, we have $Q' = 11.968$ cb. ft. Hence as $\frac{Q'}{tS} = v$, we have $\frac{11.968}{60 \times .110247} = 1.815$ ft. per second.

Mr. Provis has published in the "Transactions of Civil Engineers," vol. II. p. 203, some experiments on the flow of water through pipes $1\frac{1}{2}$ inch $= .125$ ft. in diameter; of these, No. 4, with a length of 100 ft., delivered 2 cb. ft. *per minute*, with a head of 2.5 ft. (It is presumed that the orifice of entry of the water was of the best form.) Here the velocity will be (2 cb. ft. $\div (60 \times .125^2 \times .7854) =$) 2.72 ft. per second: and hence the value of a' is found

$$\frac{2.5 \text{ ft.}}{100 \text{ ft.}} \times \frac{.125^2 \times .7854}{.125 \times 3.1415} \times \frac{1}{2.72^2} = 0.0001\,;$$

and as from these and many other experiments $a' = .0001$ we have $\frac{1}{a'} = 10{,}000$.

Substituting, then, this value of a', and solving the equation (d) for v, we have

$$v = \sqrt{\frac{h}{l} \times \frac{S}{C} \times 10000}\,;$$

or, taking the root of the factor 10,000, and placing it outside,

$$(e) \quad . \quad . \quad . \quad . \quad . \quad v = 100\sqrt{\frac{h}{l} \times \frac{S}{C}}.$$

The quantity $\frac{S}{C}$ has been called the hydraulic mean depth: it is, in the case of trapezoidal sections of channel, represented by a line de (Fig. 49); the rectangle under which, and the border $(ab + bc + cd)$, extended into one right line ad, is equal to the area of the section; the greater it is, the less the relative resistance of the surface to the volume of water passing over it.

In the case of tubes having a uniform circular section, $\frac{S}{C} = \frac{d^2 \times .7854}{d \times 3.1415} = \frac{d}{4}$, the formula ($e$) becomes then, in the case of pipes flowing full,

$$(f) \quad . \quad v = 100\sqrt{\frac{h}{l} \times \frac{d}{4}} = 50\sqrt{\frac{h}{l} \times d} \text{ ft. per sec.}$$

We have seen, in the second law of friction, that each successive couche or lamina, into which we may suppose the fluid in motion to be divided, is less and less retarded from the border towards the centre of this section: the highest velocity being consequently near the centre and in open channels at the surface. The volume of water which traverses the section of which we speak, in one second, is due to these different velocities; and the velocity, the expression for which we have now determined, is that one of these various velocities with which, if the whole section moved as one solid mass, the discharge would be the same; it is then the mean velocity, and is found in any actual experiment by dividing the volume discharged in one second by the section, as has been done in the two experiments used for determining the value of a'.

In order, then, to determine the discharge by any channel or pipe, for which we have deduced the value of v from the given inclination and hydraulic mean depth, we multiply the expressions (e) or (f) by the area. Thus from (e) we have

$$(g) \quad . \quad . \quad . \quad . \quad Q = S \times 100\sqrt{\frac{h}{l} \times \frac{S}{C}};$$

or if we put H_y for $\frac{S}{C}$, the hydraulic mean depth,

$$Q = S \times 100\sqrt{\frac{h}{l} \times H_y}.$$

And again, from (f) we have, for pipes running full,

$$Q = .7854\, d^2 \times 50 \sqrt{\frac{h}{l} \times d},$$

or

$$(h) \quad . \quad . \quad . \quad . \quad Q = 39.27 \sqrt{\frac{h}{l} \times d^5} \text{ cb. ft. per sec.}$$

In many works and reports the discharge is spoken of per minute, instead of per second: and for this unit of time we have $60 \times 39.27 = 2356.2$ as the factor outside; hence

$$Q = 2356 \sqrt{\frac{h}{l} \times d^5}$$

cb. ft. per minute, which may be written thus, $2356 \times \dfrac{\sqrt{d^5}}{\sqrt{\frac{l}{h}}}$, being the formula used by Beardmore in calculating Table 5, to which reference is again made in the Examples to this Chapter. Other formulæ for the mean velocity, generally expressed in words, are in use amongst engineers, which may be derived from those we have now given, namely, that the mean velocity of water in any pipe or channel that has attained a uniform velocity is nine-tenths of the square root of the product of twice the fall per mile into the hydraulic mean depth; or sometimes thus expressed, 0.92 into a mean proportional between twice the fall per mile and the hydraulic mean depth. These, which would not be given in words but to obviate any disadvantage arising from otherwise meeting with them for the first time, are consequences of equation (e); for the numerator and denominator of the fraction $\frac{h}{l}$ may be replaced by any numbers having the same ratio. If, then, we make $l = 5280$, the number of feet in a mile, the numerator, which we may call f, expresses the fall per mile thus, $\frac{h}{l} = \frac{f}{5280}$; and from ($e$) we have, therefore,

$$(i) \quad v = 100 \times \sqrt{\frac{f}{5280} \times H_y} = \frac{100}{72.066} \times \sqrt{f H_y} = 1.\text{[redacted]}\sqrt{f H_y};$$

and as [redacted], we have, by substituting this value in (i), [redacted]

(j) . . . . . $v =$ [redacted].

113. From the formula (h) for the discharge of pipes running full under pressure, we can, being given any two of the three quantities Q, the inclination $\frac{l}{h}$, and d, determine the other. Let it be required to find the diameter of the pipe which, with a given inclination, shall convey a given quantity of water. Dividing (h) by 39.27, and squaring both sides, we have

$$(k) \quad \dots\dots \quad \left(\frac{Q}{39.27}\right)^2 = \frac{h}{l} \times d^5,$$

and dividing by $\frac{h}{l}$, or multiplying both sides by $\frac{l}{h}$, and extracting the fifth root,

$$(l) \quad \dots\dots \quad \sqrt[5]{\left(\frac{Q}{39.27}\right)^2 \times \frac{l}{h}} = d.$$

The requisite inclination is found from (k) by dividing both sides by d^5,

$$(m) \quad \dots\dots \quad \left(\frac{Q}{39.27}\right)^2 \times \frac{1}{d^5} = \frac{h}{l};$$

and if we multiply both sides by l, we obtain h: so that if the length the water has to be conveyed be also amongst the data, we obtain the head or pressure necessary to force the given quantity along a pipe of known length and diameter,

$$(n) \quad \dots\dots \quad \left(\frac{Q}{39.27}\right)^2 \times \frac{l}{d^5} = h.$$

We cannot, however, fully determine the figure of a rectangular or trapezoidal channel from (g); solving (g) for $\frac{S^3}{C}$, we have

$$(o) \quad \dots\dots \quad \left(\frac{Q}{100}\right)^2 \times \frac{l}{h} = \frac{S^3}{C}.$$

In this we require, in addition, to be given either S or C, and also the ratio of the slopes of the sides if it be a trapezium; moreover, S and C are so related that, with given slopes, there is a maximum value of S to every given value of C; if S exceed this maximum, the solution is impossible.

114. It is found in practice that certain soils, in every excavation for whatever purpose, require a rate of slope in the banks adapted to the degree of cohesion of the ground, to obviate the danger of slips, which occur when they are too steep: this slope of the banks is, therefore, always found amongst the requisite data in the designing of channels.

In order that the banks of channels, intended to be permanent, may stand without any masonry or dry stone pitching, they should have a slope between the rates of $1\frac{1}{2}$ horizontal to 1 vertical, and 2 to 1: being made flatter according as the soil has less tenacity. In some cases even $2\frac{1}{2}$ to 1 has been adopted; the half regular hexagon has slopes of 0.58 to 1; in channels for temporary use we may have 1 to 1.

And so also is the velocity generally given; and, for the same reason, some kinds of earth being worn away, and the form of channel destroyed, by a rate which carries down the particles of the soil through which it is excavated, a velocity must therefore be assigned within this rate of motion, which has been determined experimentally for many kinds of earth.

The effect of the velocity of the water, in carrying down the particles of the ground through which the channel is excavated, depends jointly upon their tenacity and size. As to the size, we know that the cubical quantities or weights of any similar bodies decrease faster than their superficial areas; and the pressure or force urging a body down stream being, *cæteris paribus*, proportional to the transverse area, is relatively greater the less the volume; the smaller the particles, therefore, the less is the velocity required to move them. Mr. Beardmore* in Table 3 gives the following statement of the limit of *bottom* velocities in different materials:—

30 ft.	per minute	will not disturb		clay with sand and stones.
40 ft.	,,	will move along		coarse sand.
60 ft.	,,	,,	,,	fine gravel.
120 ft.	,,	,,	,,	rounded pebbles.
180 ft.	,,	,,	,,	angular stones.

The beds of rivers, protected by aquatic plants, however, bear higher velocities than this Table would assign.

Such being the natural limitations in the choice of any particular rectangle or trapezium, the engineer must proceed to determine the figure of the transverse area without violating the conditions they impose.

115. When it is desired to convey the greatest possible

* Hydraulic Tables, by N. Beardmore.

quantity of water with a given area of transverse section, then the volume discharged being directly proportional to the area, and inversely as the wetted border, we must select the figure which for a given area has the least border, or *vice versâ*.

Geometry informs us that the circle has this property: the semicircle, and therefore the semicircular channel, has the same property; the ratio between the area of the semicircle and semi-circumference being the same as that between the circle and the entire circumference. Then follow the regular demi-polygons, with less and less advantage as the number of their sides is less; and among the more practicable forms are the demi-hexagon, and finally the half-square.

As the transverse areas of artificial channels are, when without masonry, trapezoidal, the question of figure of greatest discharge is reduced to taking, among all the trapeziums with sides of a determinate slope, that which gives the greatest section for a given wetted border; or, in other words, which has the greatest hydraulic mean depth; and every different area and ratio of slopes has its particular maximum trapezium. Let p be the depth of the trapezium BF (Fig. 50), and b the bottom width BC, and $n:1$ the ratio of the slopes, or AF : FB; then the general values for S and C are

$$(p) \quad . \quad . \quad . \quad . \quad S = (b + np) \times p = bp + np^2,$$

and

$$(q) \quad . \quad . \quad . \quad . \quad . \quad C = b + 2p\sqrt{n^2 + 1}.$$

Since, then, S in the expression $\frac{S}{C}$, with slopes of $n:1$, is a maximum, its differential will be zero, and we have

$$(r) \quad . \quad . \quad . \quad . \quad pdb + bdp + 2npdp = 0;$$

and as the border is constant, its general value being differentiated, gives $db + 2dp\sqrt{n^2 + 1} = 0$. Hence $db = -2dp\sqrt{n^2 + 1}$; this being substituted in (r), gives $b = 2p(\sqrt{n^2 + 1} - n)$; with which value of b we have

$$(s) \quad . \quad . \quad . \quad . \quad \frac{S}{C} = \frac{p^2(2\sqrt{n^2 + 1} - n)}{2p(2\sqrt{n^2 + 1} - n)} = \frac{p}{2}.$$

Therefore, in all trapezoidal channels of the best form, with certain given slopes and area, the hydraulic mean depth is half the depth of the water: and hence we derive a construction for the cross section of a maximum discharging channel; remark-

ing that as $\frac{S}{C} = \frac{p}{2}$, we have $S = C \times \frac{p}{2}$. Let the trapezium, ABCD (Fig. 51) be the channel sought; from the middle point E of the top width draw lines EB and EC dividing the figure into three triangles, of which AEB and CED are identical; let EP be the perpendicular from E upon AB; then

$$\overline{AB + BC + CD} \times \frac{p}{2} = \overline{AB + CD} \times \frac{EP}{2} + BC \times \frac{p}{2};$$

and therefore $\frac{p}{2} = \frac{EP}{2}$. If, then, from E as centre, and p as radius, we describe a circle, it will touch the two sides AB and CD. If we therefore describe a circle (Fig. 52) with any radius, and draw a tangent, parallel to a horizontal diameter, produced on each side indefinitely, and between these lines draw tangents having the given inclination, we obtain a figure similar to that required, from which, by proportion, we obtain the transverse section of the channel: a construction given by Mr. Neville (Hydraulic Tables, p. 129).

116. The mean velocity of water flowing in an open channel is about 4-5ths of the maximum velocity, which is generally at the centre and upon the surface, or a little below it; and, conversely, the maximum velocity at the surface is found from the mean velocity by adding a fourth (Minard, "Cours de Construction," p. 6). Dubuat has given an empiric formula, on which have been founded Tables by most of the authors of Hydraulic Tables.

Thus, if the mean velocity be 3 ft. per sec., that at the surface is $\frac{5}{4} \times 3 = 3.75$ ft. per sec.

Also, if the observed central velocity at the surface were found to be 5.2 ft., the mean velocity is 4.16 ft. per sec.

In all the formulæ for the velocity and discharge of open channels and pipes given in this Chapter, the direction of both the pipe and channel is supposed to be in a right line: when they are curved, an additional resistance is occasioned, which diminishes the discharge, or demands an increased head to give any required discharge. This resistance is said to depend conjointly upon the square of the velocity of the water, upon the number of bends, and on the square of the sine of angle they make with the straight line of direction; and Mr. Beardmore has added, inversely, as the square root of the hydraulic mean depth; but experimenters have not been consistent in the results obtained (D'Aubuisson, §§ 196-198).

117. In the case of pipes running full, the bends may occur in the vertical plane also; and in this case the air is found to collect rapidly at the summit of such bends: air-valves must, therefore, be left to free the pipe, which may be in some cases self-acting, but are generally worked by hand at stated times. It is also necessary to proportion the diameter of the pipe in the different parts of its course, so as to make it discharge the quantity due to its diameter. Thus, in Fig. 53, which is the longitudinal section of the line of a main pipe leading the water from a reservoir at A to the point Y, if from the formation of the intervening ground it is necessary to rise up to the point B, then it is impossible for the pipe to discharge at its extremity, Y, the amount due to the diameter and *total fall* AM, from A to Y; neither can the fall from B to Y be fully efficient, because there cannot be a due supply at B. The pipe from A to B must therefore be of greater diameter than from B to Y, or than would have been necessary if the total fall from A to Y had been uniformly distributed. At the enlargement of the Edinburgh Waterworks, as designed by Mr. Jardine, the main for the first 18,300 ft. had a fall of 65 ft.; and the diameter, commencing at 20 inches, decreased to 18 inches: the remainder of the distance, 27,900 ft., had a fall of 286 ft., and a diameter of only 15 inches. The discharge into the Castle Hill distributing reservoir is only that due to the smaller diameter, laid the whole distance with an uniform fall.

118. On this main were placed, at fourteen different points,—the summits of bends in the vertical plane,—cast-iron vessels to receive the compressed air as it collected. Fig. 54 shows a vertical section of one of them, 4 ft. high, and 1.5 ft. wide, with the cock for letting off the air, which must be done every three or four days. The neglect of this precaution has been the cause of great disappointment upon the first opening of waterworks.

EXAMPLES AND PRACTICAL APPLICATIONS.

CHAPTER III.

119. (XLIV.) Calculate the area of transverse section of a channel required to convey 104 cb. ft. per sec., at a velocity of 2.5 ft. per sec. Solving the expression $Q = Sv$ for S, we have $S = \frac{Q}{v}$; hence, $\frac{104}{2.5} = 41.6$ sq. ft. for the area, S; and if together with the ratio of the sides, $n : 1$, we are given either the depth, p, or the top or bottom width, the trapezium is completely determined. For, if the depth be given, we obtain the mean width by dividing the area by the depth; and from the mean width, the top or bottom width is found by adding or subtracting $p \times n$.

Thus, if $p = 3$ and $n = 2$, or the slopes 2 to 1, the bottom width b is $\frac{41.6}{3} - (2 \times 3) = 7.866'$ ft.; and the top width, in like manner, is $\frac{41.6}{3} + (2 \times 3) = 19.866'$ ft. If the bottom width, b, be given to find p, we have $bp + np^2 = 41.6$ sq. ft. Let $b = 8$, hence, $p = \pm \sqrt{24.8} - 2 = 2.98$ ft.

It must be observed, that the inclination of the bed on the longitudinal section of the channel is not, in the above case, a matter of choice, but is absolutely determined by the data. In the former, when $p = 3$, we have $\frac{S^3}{C} = \frac{41.6^3}{21.43} = 3359.4$; and from (o) $\frac{h}{l} = \left(\frac{104}{100}\right)^2 \times \frac{1}{3359.4} = 0.000322$, or 1 in 3105, being 1.7 ft. per mile; and in the latter, $\frac{h}{l} = 0.0003204$.

To determine the dimensions of a rectangular channel formed of timber, or with vertical sides of masonry, to convey the same volume of water at the same velocity. The best form of rectangular channel being that in which the depth is half the breadth, we have the depth found from the expression—

$$p = \sqrt{\frac{Q}{2v}} = \sqrt{20.8} = 4.56 \text{ ft.},$$

and consequently the width 9.12 ft.

Calculate the dimensions of a regular half hexagon to convey the same volume of water with the same velocity. We

have the area of regular hexagon equal to $s^2 \times 2.598$, s being one of the sides. Hence—

$$s^2 \times 1.299 = 41.6, \text{ or } s = \sqrt{\frac{41.6}{1.299}} = 5.66,$$

and hence the depth $p = 4.9$ ft., and $C = 16.98$ ft.

(XLV.) Having a pipe of 2 ft. diameter, and 4000 ft. in length, with a head of 80 ft., calculate the discharge per minute. We have by (h)—

$$2356 \frac{\sqrt{2^5}}{\sqrt{\frac{4000}{80}}} = 2356 \frac{5.657}{7.071}, \text{ or } 2356 \times 0.8 = 1884.8 \text{ cb. ft. per}$$

minute.

(XLVI.) Calculate the respective discharges by pipes 1 ft. and 4 ft. diameter, the length and head being the same as in the last example.

For the pipe 1 ft. in diameter:—

$$2356 \frac{1}{\sqrt{\frac{4000}{80}}} = \frac{2356}{7.071} = 333.21 \text{ cb. ft. per minute.}$$

And for that 4 ft. in diameter:—

$$2356 \frac{\sqrt{4^5}}{7.071} = 2356 \frac{32}{7.071} = 2356 \times 4.5255 = 10{,}662 \text{ cb. ft. per}$$

minute.

Thus we find that the transverse areas in the three cases being as 1 : 4 : 16, the discharges are as 1 : 5.656 : 32; all the other data being the same.

(XLVII.) Calculate the discharge of a pipe whose area is one-half that of the 2-ft. pipe in (XLV.), and length and fall the same. The areas being as 2 : 1, the diameters are as 1.414 : 1, the $\sqrt{2}$ being = 1.414. Hence—

$$2356 \frac{\sqrt{1.414^5}}{7.071} = 792 \text{ cb. ft.};$$

and two such pipes discharge 1584 cb. ft. per minute, being 300 cb. ft. per minute less than the single 2-ft. main of equal area.

(XLVIII.) Calculate the diameter when two equal pipes give the same discharge as that of the single pipe in (XLV.). This is the same as to calculate the diameter of a pipe which shall discharge 942.4 cb. ft. per minute, with a length of 4000 ft.,

and head of 80 ft. Hence, if x be the diameter sought, we have—

$$2356 \times \frac{\sqrt{x^5}}{\sqrt{\frac{l}{h}}} = 942.4 \text{ cb. ft.}$$

And—

$$\left(\frac{942.4}{2356} \times \sqrt{50}\right)^2 = x^5;$$

or—

$$x = \sqrt[5]{(0.4 \times 7.071)^2} = \sqrt[5]{8} = \frac{\log 8}{5} = \frac{0.90309}{5} = 0.180618,$$

answering to the number 1.5157; being .1017 ft., nearly $1\frac{1}{4}$ in., greater than 1.414 ft., and discharging 150 cb. ft. more.

(XLIX.) Calculate the diameter of one large main, to convey the same quantity of water as three mains, each 2.5 ft. in diameter; the length, $2\frac{1}{2}$ miles; head, 140 ft. The quantity that may be delivered by the three mains with the above data is—

$$3 \times 2356 \frac{\sqrt{2.5^5}}{\sqrt{\frac{13200}{140}}} = 7243.3 \text{ cb. ft. per minute}$$

for $\log 2.5 = 0.39794$ and $\frac{5 \times .39794}{2} = 0.997425$, answering to the number 9.94089; also as $\sqrt{\frac{13200}{140}} = 9.7$, and $\frac{9.94089}{9.7} = 1.0248$: so that $3 \times 2356 \times 1.0248 = 7243.3$ cb. ft. per minute, as above.

Hence, the diameter of the single main being d, we have from (k)—

$$d = \sqrt[5]{\left(\frac{7243.3}{2356}\right)^2 \times 94.3} = \sqrt[5]{3.0744^2 \times 94.2},$$

or by logs $\frac{2 \times 0.48777604 + 1.9745117}{5} = 0.5900065$, answering to 3.8915 ft., about 3 ft. $10\frac{3}{4}$ in.; so that an addition of 18 in. to the diameter of one of the 30 in. mains, gives a discharge equal to all three of 30 in. diameter.

(L.) We may, from a consideration of the formula—

$$(j) \quad . \quad . \quad . \quad . \quad . \quad .9166 \sqrt{2fH_y} = v,$$

the mean velocity, explain several of the circumstances of rivers and channels. Thus, in time of floods, it is observed that the velocity is largely increased, although the slope or inclination of the surface, if it be not the same as in the ordinary and

average state, is probably even smaller in some cases. Thus, let us suppose $a' a n b b'$ (Fig. 55) to represent the transverse section of a river, at some point where it is of a regular figure, with the same slopes on each side; let a, b, be the level of the ordinary water, and a', b', that of a high flood: the transverse area, S, increases faster than the wetted border C; and therefore $\frac{S}{C}$, the hydraulic mean depth, increases faster than C, consequently the mean velocity also increases with the depth of the water, but only as the square root of the hydraulic mean depth.

Let us suppose a river 68 ft. broad at the bottom, and the slopes 2 to 1; the ordinary depth 4 ft., and the fall per mile 1.4 ft.; we have therefore—

$$v = .92 \sqrt{2 \times 1.4 \times \frac{304}{85.88}} = 2.9 \text{ ft. per sec.},$$

and the discharge is $2.9 \times 304 = 881.6$ cb. ft. per sec.

If, then, in time of flood, the river rises so as to have a depth of 7 ft., the fall per mile being supposed unaltered, we have now, consequently, the mean velocity expressed in numbers—

$$v = .92 \sqrt{2 \times 1.4 \times \frac{574}{99.304}} = 3.7 \text{ ft. per sec.}$$

And the discharge $3.7 \times 574 = 2123.8$ cb. ft.; being $2\frac{1}{2}$ times greater than the former, although the depth is but 1.75 times greater.

(LI.) It has been observed at the junction of rivers of nearly equal width and volume, that the width is not perceptibly increased below the junction: the velocity and depth are increased, so that the double volume of water is conveyed in the channel of nearly the same width; the inclination of the surface being also the same before and after the junction. Required the new depth and mean velocity of the united streams.

Let d be the depth of each of the equal streams, and b the mean width, we may assume the border to be $b + 2d$, and the hydraulic mean depth $\frac{bd}{b + 2d}$.

Thus, each river, before the junction, discharged—

$$.92 \sqrt{2f \frac{bd}{b + 2d}} \times bd,$$

and if we call x the depth of the single river after the union of the streams, and breadth b, as before, we shall have—

$$Q = .92 \sqrt{2f \times \frac{bx}{b + 2x}} \times bx = 2 \times .92 \times \sqrt{2f \times \frac{bd}{b + 2d}}\, bd.$$

Hence—

$$\sqrt{\frac{bx}{b + 2x}} \times bx = 2 \sqrt{\frac{bd}{b + 2d}} \times bd,$$

squaring both sides, and dividing out by b^3

we have $\frac{x^3}{b + 2x} = 4 \frac{d^3}{b + 2d}$,

clearing of fractions and transposing—

$$x^3 - \frac{8d^3}{b + 2d}\, x = \frac{4bd^3}{b + 2d}.$$

Let $b = 200$ ft., and $d = 5$, then—

$$x^3 - 4.762\, x - 476.2 = 0.$$

After a few substitutions we obtain $x = 8.01$ ft. If $f = 3$, the mean velocity in the rivers, before the junction, is 4.92 ft., and of the single united stream 6.14 ft.

(LII.) Calculate the quantity of water conveyed by a channel 4 ft. deep, 18 ft. wide at top, and 7 ft. at bottom, and the inclination of the surface 4 in. per mile.

The length of the side slopes is 6.8 ft.; hence the border is 20.6 ft., and the area 50 sq. ft., from which we have the hydraulic mean depth—

$$\frac{S}{C} = \frac{50}{20.6} = 2.4272 \text{ ft.}$$

Hence from (j)—

$$v = .92 \sqrt{2 \times 0.333' \times 2.4272} = 1.17 \text{ ft. per sec.},$$

and $Q = 50 \times 1.17 = 58.5$ cb. ft. per sec.

Let the slope of the above channel be doubled; required now the mean velocity and discharge, the depth and other dimensions of the transverse section remaining the same.

It is evident that the only change required in the value of the expression for v—as given in LII.—is to multiply by $\sqrt{2}$; for as the hydraulic mean depth is 2.4272, as before, we have $v = .92 \sqrt{2 \times 0.666' \times 2.427} = 1.654$ ft. per sec., and

$Q = 50 \times 1.654 = 82.7$ cb. ft. per sec.; and, generally, the discharges are, *ceteris paribus*, as the square roots of the inclinations; so that if the fall had been increased four times, the discharge would have been doubled.

(LIII.) In the mining districts of Cardiganshire it is usual to collect the supplies of water for the stamping-mills, &c., by channels, called leets, often many miles in length, and frequently deriving the water from sources altogether out of the limit of the natural watershed of the point where the power of this water is applied. The fall given to these channels is generally 1 in. per 10 fathoms, or 1 in 720, being $7.33'$ ft. per mile.

Calculate the quantity of water brought down by a leet having this fall; depth, 0.5 ft., and bottom width, 2 ft.; slopes, $1\frac{1}{2}$ to 1.

The area is 1-375 sq. ft., and the border $2 + 2 \times .5 \sqrt{1.5^2 + 1} = 3.80$ ft. Hence—

$$\frac{S}{C} = 0.362,$$

therefore the velocity is $= .92 \sqrt{2 \times 7.33' \times .362} = 2.119$ ft. per sec., and $Q = 1.375 \times 2.119 = 2.7$ cb. ft.

These streams have sometimes to be carried along the face of some nearly perpendicular rock on the side of the hill. In such places the water is conveyed in wooden troughs, supported by iron holdfasts. Calculate the transverse section of a trough to convey 2.7 cb. ft. at the above velocity. The wooden trough is rectangular, and depth half the breadth, hence the side of the square of which its transverse area is the half is

$$= \sqrt{\frac{2Q}{v}} = \sqrt{\frac{2 \times 2.7}{2.119}} = 1.6;$$

the depth, then, is 0.8 ft., and width, 1.6 ft.

(LIV.) Calculate the quantity of water conveyed by a channel of the following dimensions:—Bottom width, 10 ft.; slopes, 2 to 1; depth, 4 ft.; and fall per mile, 0.5 ft.

$$S = p^2n + pb = 4^2 \times 2 + 10 \times 4 = 72 \text{ sq. ft.},$$

$$C = b + 2p \sqrt{n^2 + 1} = 10 + 2 \times 4 \sqrt{5} = 27.9 \text{ ft.},$$

and
$$\frac{S}{C} = 2.58,$$

hence, $Q = 72 \times .92 \sqrt{2 \times .5 \times 2.58} = 106.38$ cb. ft.

If the fall had been 3 ft. per mile, then the value of Q, above found for 9.5 ft. fall, must be multiplied by $\sqrt{6} = 2.449$, and the discharge would be 260.5 cb. ft. per sec.

(LV.) Calculate the quantity of water flowing out of a tunnel by the two side drains, which were built with vertical walls of masonry; each channel being 1 ft. wide, and the water 5 inches deep; the inclination of the tunnel being 1 in 200.

$$Q = 2 \times \left\{ 0.4166' \text{sq. ft.} \times .92 \sqrt{2 \times 26.4 \times \frac{0.4166'}{1.833'}} \right\} = 2.65 \text{ cb. ft.}$$

As the depth is 0.4166′ ft., and breadth, 1 ft., the value of S is evidently 0.4166 sq. ft., and the border, $1 + 2 \times .4166 = 1.8333'$. The fall per mile for 1 in 200 is 26.4 ft.

120. The actual rate of fall adopted by engineers may be studied in the following series of examples.

The celebrated Shawswater aqueduct or channel, which conveys water to Greenock for mill-power and supply of the town, has a fall of 4 ft. per mile throughout its length of 5 miles; it discharges 43.3 cb. ft. per sec., equal to 2600 cb. ft. per minute, the depth of water being about 1.56 ft.; bottom width, 6 ft., and slopes generally, ½ to 1.

The canal from the river Durance to Marseilles, intended for irrigation and water-power, has a fall of 2.416 ft. per mile, conveying 528 cb. ft. per sec.

The Grand Ganges Canal, now opened, the largest in the world, having a course of 898 miles, navigable throughout, and furnishing irrigation to 5,400,000 acres, has a fall of 1.5 ft. per mile, a depth of 10 ft., and a constant bottom width of 140 ft.; and is intended to convey 6750 ft. per sec.; the slopes are not mentioned, but at 1½ to 1, the formula would give 7316 cb. ft. The fall in this canal may, from its vast dimensions of transverse section, be considered very great, and the resulting velocity such as would ordinarily wear down the sides—in tropical countries, however, it would appear that the growth of aquatic plants, causing a great increase of resistance on the bottom and sides, is a serious evil to canal works, and necessitates a greater inclination of surface than would be required in higher latitudes.—("Italian Irrigation," by Captain B. Smith.)

Covered channels constructed of masonry offer less resistance from the greater truth of the lines of the work, and evenness of surface of bottom and sides of channel, when well constructed, than is found in open channels excavated in the ground.

The Croton Waterworks, supplying the city of New York, afford the finest modern example of water supply having this great advantage of a covered channel, so essential to the purity and low temperature of the water. The transverse section of the culvert is given, with figured dimensions, in Fig. 55. The fall is $13\frac{1}{2}$ inches, or 1.125 ft. per mile; the discharge is said to be 60,000,000 gallons in twenty-four hours, which gives 111.1 cb. ft. per sec.

The projected supply from Loch Katrine for the city of Glasgow is to be conveyed by a culvert or tunnel 8 ft. in diameter, and with a slope of 1 in 6336; in crossing the intervening valleys, a cast-iron pipe, 4 ft. in diameter, is used, like an inverted syphon; the total fall from point of entry to that of delivery being at the rate of 1 in 1000, or 5.28 ft. per mile.

The Canal de l'Ourcq, conveying water from the Ourcq to Paris, has an inclination of 0.0001056, or 33.26 ft. fall in the total length of 314,966 ft. The gauging of the river l'Ourcq showed that 106.61 cb. ft. were to be conveyed down; the projected navigation required a depth of 4.9213 ft., and it was deemed necessary to give a velocity to the current of 1.1483 ft. per sec. From these data it resulted, from the use of the complicated formula which introduced the simple power of the velocity (p. 108), and such other modifications of it as were assumed to represent every different kind of resistance, that the inclination should be 0.00005502. But the engineer, fearing the retarding effect of the weeds and plants which grow abundantly on the bottom and sides, finally adopted that named above, 0.0001056, or nearly double that given by calculation, making the fall 0.5575 ft. per mile.

The New River (as it is still called), conveying water to London from near Ware on the river Lea, is graduated to 0.25 ft. per mile; having a winding couse of 39 miles; its breadth is said to be 18 ft., and depth, 4 ft. The fall in this canal, intended for domestic supply, is too small, as in summer the water becomes raised in temperature from the great surface exposed in its long course, and from the slow rate of motion, which is about half a mile per hour.

THE END.

A CATALOG

OF

NEW WORKS IN GENERAL LITERATURE,

PUBLISHED BY

LONGMAN, BROWN, GREEN, AND LONGMANS,

39, PATERNOSTER ROW, LONDON.

CLASSIFIED INDEX.

Agriculture and Rural Affairs.

Pages

Bayldon On valuing Rents, &c. - 4
Caird's Letters on Agriculture - 5
Cecil's Stud Farm - - - 6
Loudon's Agriculture - - - 13
" Self-Instruction - - 13
" Lady's Country Compan. 14
Low's Elements of Agriculture - 14
" Domesticated Animals - 14

Arts, Manufactures, and Architecture.

Bourne's Catechism of the Steam Engine - - - - - 4
" On the Screw Propeller - 4
Brande's Dictionary of Science, &c 4
" Organic Chemistry - - 4
Chevreul on Colour - - - - 6
Cresy's Civil Engineering - - 6
Eastlake On Oil Painting - - 7
Gwilt's Encyclo. of Architecture - 8
Jameson's Sacred & Legendary Art 10
" Commonplace Book - 10
Konig's Pictorial Life of Luther - 8
Loudon's Rural Architecture - 13
Moseley's Engineering - - - 16
Richardson's Art of Horsemanship 18
Steam Engine, by the Artisan Club 4
Tate on Strength of Materials - 21
Ure's Dictionary of Arts, &c. - 22

Biography.

Bodenstedt and Wagner's Schamyl 24
Brightwell's Memorials of Opie - 17
Bunsen's Hippolytus - - - 5
Chesterton's Autobiography - - 6
Clinton's (Fynes) Autobiography 6
Cockayne's Marshal Turenne - 24
Freeman's Life of Kirby - 11
Haydon's Autobiography, by Taylor 8
Holcroft's Memoirs - - - 24
Holland's (Lord) Memoirs - - 9
Lardner's Cabinet Cyclopædia - 12
Maunder's Biographical Treasury - 15
Memoir of the Duke of Wellington 24
Memoirs of James Montgomery - 15
Merivale's Memoirs of Cicero - 15
Russell's Memoirs of Moore - - 16
" Life of Lord Wm. Russell 19
Southey's Life of Wesley - - 20
" Life and Correspondence 20
Stephen's Ecclesiastical Biography 21
Taylor's Loyola - - - - 21
" Wesley - - - - 21
Townsend's Eminent Judges - 22
Waterton's Autobiography & Essays 22

Books of General Utility.

Acton's Cookery - - - - 3
Black's Treatise on Brewing - - 4
Cabinet Gazetteer - - - - 5
" Lawyer - - - - 5
Cust's Invalid's Own Book - 6
Hints on Etiquette - - - 9
Hudson's Executor's Guide - - 10
" On Making Wills - - 10
Lardner's Cabinet Cyclopædia - 12
Loudon's Self-Instruction - - 13
" Lady's Companion - 14
" Amateur Gardener 13
Maunder's Treasury of Knowledge 15
" Biographical Treasury 15
" Scientific Treasury - 15
" Treasury of History - 15
" Natural History - - 15
Pocket and the Stud - - - 8
Pycroft's English Reading - - 18
Reece's Medical Guide - - - 18

Pages.

Rich's Comp. to Latin Dictionary 18
Richardson's Art of Horsemanship 18
Riddle's Latin Dictionaries - - 18
Roget's English Thesaurus - - 19
Rowton's Debater - - - - 19
Short Whist - - - - - 20
Thomson's Interest Tables - - 22
Traveller's Library - 23 & 24
Webster's Domestic Economy - 22
Willich's Popular Tables - - 24
Wilmot's Abridgment of Blackstone's Commentaries - - 24

Botany and Gardening.

Conversations on Botany - - 6
Hooker's British Flora - - - 9
" Guide to Kew Gardens - 9
Lindley's Introduction to Botany 11
" Theory of Horticulture - 11
Loudon's Hortus Britannicus - 13
" Amateur Gardener - 13
" Self-Instruction - - 13
" Trees and Shrubs - - 13
" Gardening - - - 13
" Plants - - 13
Rivers's Rose Amateur's Guide - 18

Chronology.

Blair's Chronological Tables - 4
Bunsen's Ancient Egypt - - 5
Haydn's Beatson's Index - - 8
Nicolas's Chronology of History - 12

Commerce and Mercantile Affairs.

Atkinson's Shipping Laws - - 3
Francis On Life Assurance - - 8
Loch's Sailor's Guide - - - 13
Lorimer's Letters to a Young Master Mariner - - - - 13
M'Culloch's Commerce & Navigation 14
Thomson's Interest Tables - - 22

Criticism, History, and Memoirs.

Austin's Germany - - - - 3
Balfour's Sketches of Literature - 3
Blair's Chron. and Histor. Tables - 4
Bunsen's Ancient Egypt - - 5
" Hippolytus - - - 5
Burton's History of Scotland - 5
Chalybaeus's Modern Speculative Philosophy - - - - - 6
Conybeare and Howson's St. Paul 6
Eastlake's History of Oil Painting 7
Erskine's History of India - - 7
Francis's Annals of Life Assurance 7
Gleig's Leipsic Campaign - - 24
Gurney's Historical Sketches - 8
Hamilton's Essays from the Edinburgh Review - - - - - 8
Haydon's Autobiography, by Taylor 8
Holland's (Lord) Foreign Reminiscences - - - - 9
" Whig Party - 9
Jeffrey's (Lord) Contributions - 10
Kemble's Anglo-Saxons - 11
Lardner's Cabinet Cyclopædia - 12
Macaulay's Crit. and Hist. Essays 14
" History of England - 14
" Speeches - - - 14
Mackintosh's Miscellaneous Works 14
" History of England - 14
M'Culloch's Geographical Dictionary 14
Martineau's Church History - - 15
Maunder's Treasury of History - 15
Memoir of the Duke of Wellington 24
Merivale's History of Rome - - 15
" Roman Republic - 15

Pages

Milner's Church History - - 15
Moore's (Thomas) Memoirs, &c. - 16
Mure's Greek Literature - 16
Ranke's Ferdinand & Maximilian 24
Rich's Comp. to Latin Dictionary 18
Riddle's Latin Dictionaries - 18
Rogers's Essays from the Edinburgh Review - - - - - - 19
Roget's English Thesaurus - - 19
Russell's (Lady Rachel) Letters - 19
" Life of Lord W. Russell 19
St. John's Indian Archipelago - 19
Schmitz's History of Greece - 19
Smith's Sacred Annals - - - 20
Southey's The Doctor &c - - 21
Stephen's Ecclesiastical Biography 21
" Lectures on French History 21
Sydney Smith's Works - - - 20
" Select Works - 24
" Lectures - - 20
Taylor's Loyola - - - - 21
" Wesley - - - - 21
Thirlwall's History of Greece - 21
Townsend's State Trials - - 22
Turkey and Christendom - - 24
Turner's Anglo Saxons - - 22
" Middle Ages - - 22
" Sacred Hist. of the World 22
Zumpt's Latin Grammar - - 24

Geography and Atlases.

Butler's Geography and Atlases - 5
Cabinet Gazetteer - - - - 5
Durrieu's Morocco - - - 24
Hall's Large Library Atlas - - 8
Hughes's Australian Colonies - 24
Jesse's Russia and the War - - 10
Johnston's General Gazetteer - 11
M'Culloch's Geographical Dictionary 14
" Russia and Turkey - 24
Milner's Baltic Sea - - - 15
Murray's Encyclo. of Geography - 17
Sharp's British Gazetteer - - 19
Wheeler's Geography of Herodotus 24

Juvenile Books.

Amy Herbert - - - - - 19
Corner's Children's Sunday Book 6
Earl's Daughter (The) - - - 19
Experience of Life - - 20
Gertrude - - - - - 19
Howitt's Boy's Country Book - 10
" (Mary) Children's Year - 10
Katharine Ashton - - - 20
Lady Una and her Queendom - 11
Laneton Parsonage - - - 19
Mrs. Marcet's Conversations - - 15
Margaret Percival - - - - 20
Pycroft's English Reading - - 18

Medicine and Surgery.

Bull's Hints to Mothers - - - 4
" Management of Children - 4
Copland's Dictionary of Medicine - 6
Cust's Invalid's Own Book - - 6
Holland's Mental Physiology - 9
Latham On Diseases of the Heart - 11
Little On Treatment of Deformities 11
Moore On Health, Disease, & Remedy 16
Pereira On Food and Diet - 17
Psychological Inquiries - - 18
Reece's Medical Guide - - - 18

Miscellaneous and General Literature.

Atkinson's Sheriff-Law - - 3
Austin's Sketches of German Life 3
Carlisle's Lectures and Addresses 24

CLASSIFIED INDEX.

Pages.

Chalybaeus's Modern Speculative Philosophy - 6
Defence of *Eclipse of Faith* - 7
Eclipse of Faith - 7
Greg's Essays on Political and Social Science - 8
Haydn's Book of Dignities - 8
Hole's Essay on Mechanics' Institutions - 9
Holland's Mental Physiology - 9
Hooker's Kew Guide - 9
Howitt's Rural Life of England - 9
" Visits to Remarkable Places 9
Jameson's Commonplace Book - 10
Jeffrey's (Lord) Contributions - 10
Last of the Old Squires - 17
Loudon's Lady's Companion - 14
Macaulay's Crit. and Hist. Essays 14
" Speeches - 14
Mackintosh's Miscellaneous Works 14
Memoirs of a Maître-d'Armes - 24
Maitland's Church in the Catacombs 14
Pascal's Works, by Pearce - 17
Pycroft's English Reading - 18
Rich's Comp. to Latin Dictionary 18
Riddle's Latin Dictionaries - 18
Rowton's Debater - 19
Seaward's Narrative of his Shipwreck 19
Sir Roger de Coverley - 20
Smith's (Rev. Sydney) Works - 21
Southey's Common-place Books - 21
" The Doctor &c. - 21
Souvestre's Attic Philosopher - 24
" Confessions of a Working Man 24
Stephen's Essays - 21
Stow's Training System - 21
Thomson's Laws of Thought - 21
Townsend's State Trials - 22
Willich's Popular Tables - 24
Yonge's English-Greek Lexicon - 24
" Latin Gradus - 24
Zumpt's Latin Grammar - 24

Natural History in general.

Catlow's Popular Conchology - 6
Ephemera and Young On the Salmon 7
Gosse's Nat. Hist. of Jamaica - 8
Kemp's Natural Hist. of Creation 24
Kirby and Spence's Entomology - 11
Lee's Elements of Natural History 11
Maunder's Natural History - 15
Turton's Shells of the British Islands 22
Waterton's Essays on Natural Hist. 22
Youatt's The Dog - 24
" The Horse - 24

1-Volume Encyclopædias and Dictionaries.

Blaine's Rural Sports - 4
Brande's Science, Literature, & Art 4
Copland's Dictionary of Medicine - 6
Cresy's Civil Engineering - 6
Gwilt's Architecture - 8
Johnston's Geographical Dictionary 11
Loudon's Agriculture - 13
" Rural Architecture - 13
" Gardening - 13
" Plants - 13
" Trees and Shrubs - 13
M'Culloch's Geographical Dictionary 14
" Dictionary of Commerce 14
Murray's Encyclo. of Geography - 17
Sharp's British Gazetteer - 19
Ure's Dictionary of Arts, &c. - 22
Webster's Domestic Economy - 22

Religious & Moral Works.

Amy Herbert - 19
Atkinson On the Church - 3
Bloomfield's Greek Testament - 4
" Annotations on do. - 4
Calvert's Wife's Manual - 6
Conybeare and Howson's St. Paul 6
Corner's Sunday Book - 6
Dale's Domestic Liturgy - 7
Defence of *Eclipse of Faith* - 7
Discipline - 7
Earl's Daughter (The) - 19
Eclipse of Faith - 7
Englishman's Greek Concordance 7
Englishman's Heb. & Chald. Concord. 7
Experience of Life (The) - 20
Gertrude - 19
Harrison's Light of the Forge - 8
Hook's Lectures on Passion Week 9
Horne's Introduction to Scriptures 9
" Abridgment of ditto - 9
Hulbert on Job - 10
Jameson's Sacred Legends - 10
" Monastic Legends - 10
" Legends of the Madonna 10
Jeremy Taylor's Works - 10
Katharine Ashton - 20
Kippis's Hymns - 11
König's Life of Luther - 8
Lady Una and her Queendom - 11
Laneton Parsonage - 19
Letters to My Unknown Friends - 11
" on Happiness - 11
Litton's Church of Christ - 13
Maitland's Church in the Catacombs 14
Margaret Percival - 20
Martineau's Church History - 15
Milner's Church of Christ - 15
Montgomery's Original Hymns - 16
Moore On the Use of the Body - 16
" " Soul and Body - 16
" 's Man and his Motives - 16
Mormonism - 24
Neale's Closing Scene - 17
" Resting-Places of the Just 17
" Riches that Bring no Sorrow - 17
" Risen from the Ranks - 17
Newman's (J. H.) Discourses - 17
Ranke's Ferdinand & Maximilian 24
Readings for Lent - 20
" Confirmation - 20
Robinson's Lexicon to the Greek Testament - 18
Saints our Example - 19
Self-Denial - 19
Sermon in the Mount - 19
Sermon on the Mount illuminated 19
Sinclair's Journey of Life - 20
Smith's (Sydney) Moral Philosophy 20
" (G.) Sacred Annals - 20
Southey's Life of Wesley - 20
Stephen's Ecclesiastical Biography 21
Taylor's Loyola - 21
" Wesley - 21
Theologia Germanica - 21
Thumb Bible (The) - 22
Turner's Sacred History - 22

Poetry and the Drama.

Arnold's Poems - 3
Aikin's (Dr.) British Poets - 3
Baillie's (Joanna) Poetical Works 3
Baxter's Iliad of Homer - 3
Bode's Ballads from Herodotus - 4
Calvert's Wife's Manual - 5
Flowers and their kindred Thoughts 17
Goldsmith's Poems, illustrated - 8
Kent's Aletheia - 11
Kippis's Hymns - 11
L. E. L.'s Poetical Works - 11
Linwood's Anthologia Oxoniensis - 11
Macaulay's Lays of Ancient Rome 14
Montgomery's Poetical Works - 16
" Original Hymns - 16
Moore's Poetical Works - 16
" Lalla Rookh - 16
" Irish Melodies - 16
" Songs and Ballads - 16
Shakspeare, by Bowdler - 20
" Sentiments & Similes 10
Southey's Poetical Works - 21
" British Poets - 21
Thomson's Seasons, illustrated - 22
Thornton's Zohrab - 22
Watts's Lyrics of the Heart - 22

Political Economy and Statistics.

Banfield's Statistical Companion - 4
Caird's Letters on Agriculture - 5
Francis On Life Assurance - 7
Greg's Essays on Political and Social Science - 8
Laing's Notes of a Traveller - 11 & 24
M'Culloch's Geog. Statist. &c. Dict. 14
" Dictionary of Commerce 14
" London - 24
" Statistics of Gt. Britain 14
Marcet's Political Economy - 15
Willich's Popular Tables - 24

The Sciences in General and Mathematics.

Bourne's Catechism of the Steam Engine - 4
" On the Screw Propeller - 4
Brande's Dictionary of Science, &c. 4
" Lectures on Organic Chemistry 4
Cresy's Civil Engineering - 6
De la Beche's Geology of Cornwall, &c. 7
" Geological Observer - 7
De la Rive's Electricity - 7
Faraday's Non-Metallic Elements 7
Fullom's Marvels of Science - 7
Herschel's Outlines of Astronomy 9
Holland's Mental Physiology - 9
Humboldt's Aspects of Nature - 10
" Cosmos - 10
Hunt On Light - 10
Lardner's Cabinet Cyclopædia - 12
Marcet's (Mrs.) Conversations - 15
Moseley's Engineering & Architecture 16
Owen's Lectures on Comp. Anatomy 17
Our Coal Fields and our Coal Pits 24
Peschel's Elements of Physics - 17
Phillips's Fossils of Cornwall, &c. 18
" Mineralogy - 18
" Guide to Geology - 17
Portlock's Geology of Londonderry 18
Smee's Electro-Metallurgy - 20
Steam Engine (The) - 4
Tate On Strength of Materials - 21
Todd's Tables of Circles - 22
Wilson's Electric Telegraph - 24

Rural Sports.

Baker's Rifle and Hound in Ceylon 3
Berkeley's Reminiscences - 4
Blaine's Dictionary of Sports - 4
Cecil's Stable Practice - 6
" Records of the Chase - 6
" Stud Farm - 6
The Cricket-Field - 6
Ephemera On Angling - 7
" Book of the Salmon - 7
The Hunting Field - 8
Loudon's Lady's Country Comp. - 14
Pocket and the Stud - 8
Practical Horsemanship - 8
Pulman's Fly Fishing - 18
Richardson's Horsemanship - 18
St. John's Sporting Rambles - 19
Stable Talk and Table Talk - 8
Stonehenge On the Greyhound 21
The Stud, for Practical Purposes - 8

Veterinary Medicine, &c.

Cecil's Stable Practice - 6
" Stud Farm - 6
Hunting Field (The) - 8
Morton's Veterinary Pharmacy 16
Pocket and the Stud - 8
Practical Horsemanship - 8
Richardson's Horsemanship - 18
Stable Talk and Table Talk - 8
Stud (The) - 8
Youatt's The Dog - 24
" The Horse - 24

Voyages and Travels.

Baker's Rifle and Hound in Ceylon 3
Barrow's Continental Tour - 24
Carlisle's Turkey and Greece - 5
De Custine's Russia - 24
Eothen - 24
Ferguson's Swiss Travels - 24
Forester and Biddulph's Norway - 7
Gironière's Philippines - 24
Hill's Travels in Siberia - 9
Hope's Brittany and the Bible - 24
" Chase in Brittany - 24
Howitt's Art-Student in Munich - 9
Huc's Tartary, Thibet, and China 24
Hughes's Australian Colonies - 24
Humbley's Indian Journal - 10
Humboldt's Aspects of Nature - 10
Jameson's Canada - 24
Jerrmann's St. Petersburg - 24
Laing's Norway - 24
" Notes of a Traveller 11 & 24
Macintosh's Turkey and Black Sea 14
Oldmixon's Piccadilly to Pera - 17
Osborn's Arctic Journal - 17
Peel's Nubian Desert - 17
Pfeiffer's Voyage round the World 24
Power's New Zealand Sketches - 18
Richardson's Arctic Boat Voyage 18
Seaward's Narrative - 19
St. John's (H.) Indian Archipelago 19
" (J. A.) Isis - 19
" " There & Back again 19
" (Hon. F.) Rambles - 19
Sutherland's Arctic Voyage - 21
Traveller's Library - 23 & 24
Werne's African Wanderings - 24

Works of Fiction.

Arnold's Oakfield - 3
Lady Willoughby's Diary - 24
Macdonald's Villa Verocchio - 14
Sir Roger de Coverley - 20
Southey's The Doctor &c. - 21

ALPHABETICAL CATALOGUE

OF

NEW WORKS AND NEW EDITIONS

PUBLISHED BY

Messrs. LONGMAN, BROWN, GREEN, and LONGMANS,

PATERNOSTER ROW, LONDON.

Miss Acton's Modern Cookery-Book.—Modern Cookery in all its Branches, reduced to a System of Easy Practice. For the use of Private Families. In a Series of Receipts, all of which have been strictly tested, and are given with the most minute exactness. By ELIZA ACTON. New Edition; with various Additions, Plates and Woodcuts. Fcp. 8vo. price 7s. 6d.

Aikin.—Select Works of the British Poets, from Ben Jonson to Beattie. With Biographical and Critical Prefaces by Dr. AIKIN. New Edition, with Supplement by LUCY AIKIN; consisting of additional Selections from more recent Poets. 8vo. price 18s.

Arnold.—Poems. By Matthew Arnold. Second Edition. Fcp. 8vo. price 5s. 6d.

Arnold.—Oakfield; or, Fellowship in the East. By W. D. ARNOLD, Lieutenant 58th Regiment, Bengal Native Infantry. The Second Edition, revised. 2 vols. post 8vo. price 21s.

Atkinson (G.)—Sheriff-Law; or, a Prac-tical Treatise on the Office of Sheriff, Undersheriff, Bailiffs, &c.: Their Duties at the Election of Members of Parliament and Coroners, Assizes, and Sessions of the Peace; Writs of Trial; Writs of Inquiry; Compensation Notices; Interpleader; Writs; Warrants; Returns; Bills of Sale; Bonds of Indemnity, &c. By GEORGE ATKINSON, Serjeant-at-Law. Third Edition, revised. 8vo. price 10s. 6d.

Atkinson (G.)—The Shipping Laws of the British Empire: Consisting of *Park on Marine Assurance*, and *Abbott on Shipping*. Edited by GEORGE ATKINSON, Serjeant-at-Law. 8vo. price 10s. 6d.

Atkinson (W.)—The Church: An Expla-nation of the Meaning contained in the Bible; shewing the Ancient, Continued, and Prevailing Error of Man, the Substitution of Worship for Religion; and shewing that the Principles of all Right Individual Action and of General Government, or the Government of all Nations, are comprised in Revealed Religion. By WILLIAM ATKINSON. 2 vols. 8vo. price 30s.

Austin.—Germany from 1760 to 1814; Or, Sketches of German Life from the Decay of the Empire to the Expulsion of the French. By Mrs. AUSTIN. Post 8vo. price 12s.

Joanna Baillie's Dramatic and Poetical Works, complete in One Volume: Comprising the Plays of the Passions, Miscellaneous Dramas, Metrical Legends, Fugitive Pieces (several now first published), and Ahalya Baee. Second Edition, including a new Life of Joanna Baillie; with a Portrait, and a View of Bothwell Manse. Square crown 8vo. price 21s. cloth; or 42s. bound in morocco.

Baker.—The Rifle and the Hound in Ceylon. By S. W. BAKER, Esq. With several Illustrations printed in Colours, and Engravings on Wood. 8vo. price 14s.

Balfour.—Sketches of English Literature from the Fourteenth to the Present Century. By CLARA LUCAS BALFOUR. Fcp. 8vo. 7s.

Barter. — Homer's Iliad, translated almost literally into the Spenserian Stanza; with Notes. By W. G. T. BARTER. 8vo. price 18s.

Banfield.—The Statistical Companion for 1854: Exhibiting the most Interesting Facts in Moral and Intellectual, Vital, Economical, and Political Statistics, at Home and Abroad. Corrected to the Present Time; and including the Census of the British Population taken in 1851. Compiled from Official and other Authentic Sources, by T. C. BANFIELD, Esq. Fcp. 8vo. price 6s.

Bayldon's Art of Valuing Rents and Tillages, and Tenant's Right of Entering and Quitting Farms, explained by several Specimens of Valuations; with Remarks on the Cultivation pursued on Soils in different Situations. Adapted to the Use of Landlords, Land-Agents, Appraisers, Farmers, and Tenants. New Edition; corrected and revised by JOHN DONALDSON. 8vo. 10s. 6d.

Berkeley.—Reminiscences of a Huntsman. By the Honourable GRANTLEY F. BERKELEY. With Four Etchings by John Leech (one coloured). 8vo. price 14s.

Black's Practical Treatise on Brewing, Based on Chemical and Economical Principles: With Formulæ for Public Brewers, and Instructions for Private Families. New Edition, with Additions. 8vo. price 10s. 6d.

Blaine's Encyclopædia of Rural Sports; Or, a complete Account, Historical, Practical, and Descriptive, of Hunting, Shooting, Fishing, Racing, and other Field Sports and Athletic Amusements of the present day. A new and thoroughly revised Edition: The Hunting, Racing, and all relative to Horses and Horsemanship, revised by HARRY HIEOVER; Shooting and Fishing by EPHEMERA; and Coursing by Mr. A. GRAHAM. With upwards of 600 Wood Engravings. 8vo. price 50s. half-bound.

Blair's Chronological and Historical Tables, from the Creation to the present time: With Additions and Corrections from the most authentic Writers; including the Computation of St. Paul, as connecting the Period from the Exode to the Temple. Under the revision of Sir HENRY ELLIS, K.H. New Edition, with Corrections. Imperial 8vo. price 31s. 6d. half-morocco.

Bloomfield.—The Greek Testament: With copious English Notes, Critical, Philological, and Explanatory. Especially formed for the use of advanced Students and Candidates for Holy Orders. By the Rev. S. T. BLOOMFIELD, D.D., F.S.A. New Edition. 2 vols. 8vo. with Map, price £2.

Dr. Bloomfield's Additional Annotations on the above. 8vo. price 15s.

Bloomfield.—College and School Greek Testament: With brief English Notes, chiefly Philological and Explanatory, especially formed for use in Colleges and the Public Schools. By the Rev. S. T. BLOOMFIELD, D.D., F.S.A. Seventh *and cheaper* Edition, improved; with Map and Index. Fcp. 8vo. price 7s. 6d.

Dr. Bloomfield's College and School Lexicon to the Greek Testament. Fcp. 8vo. price 10s. 6d.

Bode.—Ballads from Herodotus: With an Introductory Poem. By the Rev. J. E. BODE, M.A., late Student of Christ Church. 16mo. price 5s.

Bourne.—A Treatise on the Steam Engine, in its Application to Mines, Mills, Steam Navigation, and Railways. By the Artisan Club. Edited by JOHN BOURNE, C.E. New Edition; with 30 Steel Plates and 349 Wood Engravings. 4to. price 27s.

Bourne.—A Catechism of the Steam Engine, illustrative of the Scientific Principles upon which its Operation depends, and the Practical Details of its Structure, in its applications to Mines, Mills, Steam Navigation, and Railways: With various Suggestions of Improvement. By JOHN BOURNE, C.E. New Edition. Fcp. 8vo. price 6s.

Bourne.—A Treatise on the Screw Propeller: With various Suggestions of Improvement. By JOHN BOURNE, C.E., Editor of *The Artisan Club's Treatise on the Steam Engine*. With 20 large Plates and numerous Woodcuts. 4to. price 38s.

Brande.—A Dictionary of Science, Literature, and Art; comprising the History, Description, and Scientific Principles of every Branch of Human Knowledge; with the Derivation and Definition of all the Terms in General Use. Edited by W. T. BRANDE, F.R.S.L. and E.; assisted by Dr. J. CAUVIN. The Second Edition, revised and corrected; including a Supplement, and numerous Wood Engravings. 8vo. 60s.

Professor Brande's Lectures on Organic Chemistry, as applied to Manufactures, including Dyeing, Bleaching, Calico-Printing, Sugar-Manufacture, the Preservation of Wood, Tanning, &c. delivered before the Members of the Royal Institution in the Session of 1852. Arranged by permission from the Lecturer's Notes by J. SCOFFERN, M.B. Fcp. 8vo. [*Just ready.*

Bull. — The Maternal Management of Children in Health and Disease. By T. Bull, M.D., Member of the Royal College of Physicians; formerly Physician-Accoucheur to the Finsbury Midwifery Institution. New Edition. Fcp. 8vo. price 5s.

Bull.—Hints to Mothers, for the Management of their Health during the Period of Pregnancy and in the Lying-in Room: With an Exposure of Popular Errors in connexion with those subjects, &c.; and Hints upon Nursing. By T. Bull, M.D. New Edition. Fcp. price 5s.

Bunsen. — Christianity and Mankind, their Beginnings and Prospects. By Christian Charles Josias Bunsen, D.D., D.C.L., D.Ph. Being a New Edition, corrected, remodelled, and extended, of *Hippolytus and his Age.* 7 vols. 8vo. price £5. 5s.

*** This Second Edition of the *Hippolytus* is composed of three distinct works, which may be had separately, as follows:—

Historical Section.

1. Hippolytus and his Age; or, the Beginnings and Prospects of Christianity. 2 vols. 8vo. price £1. 10s.
   - I. Hippolytus and the Teachers of the Apostolical Age;
   - II. The Life of the Christians of the Apostolical Age.

Philological Section.

2. Outline of the Philosophy of Universal History applied to Language and Religion. 2 vols. 8vo. price £1. 13s.

Philosophical Section.

3. Analecta Ante-Nicaena. 3 vols. 8vo. price £2. 2s.
   - I. Reliquiae Literariae;
   - II. Reliquiae Canonicae;
   - III. Reliquiae Liturgicae: Cum Appendicibus ad Tria Analectorum Volumina.

Bunsen. — Egypt's Place in Universal History: An Historical Investigation, in Five Books. By C. C. J. Bunsen, D.D. D.C.L., D.Ph. Translated from the German, by C. H. Cottrell, Esq. M.A. Vol. I. with many Illustrations. 8vo. price 28s.

*** The Second Volume is preparing for publication.

Burton.—The History of Scotland, from the Revolution to the Extinction of the last Jacobite Insurrection (1689—1748). By John Hill Burton, Author of *The Life of David Hume,* &c. 2 vols. 8vo. price 26s.

Bishop Butler's General Atlas of Modern and Ancient Geography; comprising Fifty-two full-coloured Maps; with complete Indices. New Edition, nearly all re-engraved, enlarged, and greatly improved; with Corrections from the most authentic sources in both the Ancient and Modern Maps, many of which are entirely new. Edited by the Author's Son. Royal 4to. 24s. half-bound.

Separately { The Modern Atlas of 28 full-coloured Maps. Rl. 8vo. 12s.
The Ancient Atlas of 24 full-coloured Maps. Rl. 8vo. 12s. }

Bishop Butler's Sketch of Modern and Ancient Geography. New Edition, carefully revised, with such Alterations introduced as continually progressive Discoveries and the latest Information have rendered necessary. 8vo. price 9s.

The Cabinet Gazetteer: A Popular Exposition of all the Countries of the World; their Government, Population, Revenues, Commerce, and Industries; Agricultural, Manufactured, and Mineral Products; Religion, Laws, Manners, and Social State: With brief Notices of their History and Antiquities. From the latest Authorities. By the Author of *The Cabinet Lawyer.* Fcp. 8vo. price 10s. 6d. cloth; or 13s. calf lettered.

The Cabinet Lawyer: A Popular Digest of the Laws of England, Civil and Criminal; with a Dictionary of Law Terms, Maxims, Statutes, and Judicial Antiquities; Correct Tables of Assessed Taxes, Stamp Duties, Excise Licenses, and Post-Horse Duties; Post-Office Regulations, and Prison Discipline. 16th Edition, comprising the Public Acts of the Session 1853. Fcp. 8vo. 10s. 6d.

Caird.—English Agriculture in 1850 and 1851; Its Condition and Prospects. By James Caird, Esq., of Baldoon, Agricultural Commissioner of *The Times.* The Second Edition. 8vo. price 14s.

Calvert. — The Wife's Manual; or, Prayers, Thoughts, and Songs on Several Occasions of a Matron's Life. By the Rev. William Calvert, Rector of St. Antholin, and one of the Minor Canons of St. Paul's. Printed by C. Whittingham; and ornamented from Designs by the Author in the style of *Queen Elizabeth's Prayer Book.* Crown 8vo. price 10s. 6d.

Carlisle (Lord).—A Diary in Turkish and Greek Waters. By the Right Hon. the Earl of Carlisle. Post 8vo.

[*Nearly ready*

Catlow.—Popular Conchology; or, the Shell Cabinet arranged according to the Modern System: With a detailed Account of the Animals; and a complete Descriptive List of the Families and Genera of Recent and Fossil Shells. By AGNES CATLOW. Second Edition, much improved; with 405 Woodcut Illustrations. Post 8vo. price 14s.

Cecil.—The Stud Farm; or, Hints on Breeding Horses for the Turf, the Chase, and the Road. Addressed to Breeders of Race Horses and Hunters, Landed Proprietors, and especially to Tenant Farmers. By CECIL. Fcp. 8vo. with Frontispiece, 5s.

Cecil's Records of the Chase, and Memoirs of Celebrated Sportsmen; Illustrating some of the Usages of Olden Times and comparing them with prevailing Customs: Together with an Introduction to most of the Fashionable Hunting Countries; and Comments. With Two Plates by B. Herring. Fcp. 8vo. price 7s. 6d. half-bound.

Cecil's Stable Practice; or, Hints on Training for the Turf, the Chase, and the Road; with Observations on Racing and Hunting, Wasting, Race Riding, and Handicapping: Addressed to owners of Racers, Hunters, and other Horses, and to all who are concerned in Racing, Steeple Chasing, and Fox Hunting. Fcp. 8vo. with Plate, price 5s. half-bound.

Chalybaeus's Historical Survey of Modern Speculative Philosophy, from Kant to Hegel: Designed as an Introduction to the Opinions of the Recent Schools. Translated from the German by ALFRED TULK. Post 8vo. price 8s. 6d.

Captain Chesterton's Autobiography.— Peace, War, and Adventure: Being an Autobiographical Memoir of George Laval Chesterton, formerly of the Field-Train Department of the Royal Artillery, subsequently a Captain in the Army of Columbia, and at present Governor of the House of Correction at Cold Bath Fields. 2 vols. post 8vo. price 16s.

Chevreul on Colour.—The Principles of Harmony and Contrast of Colours, and their Applications to the Arts: Including Painting, Interior Decoration, Tapestries, Carpets, Mosaics, Coloured Glazing, Paper-Staining, Calico Printing, Letterpress Printing, Map Colouring, Dress, Landscape and Flower Gardening, &c. By M. E. CHEVREUL, Membre de l'Institut de France, etc. Translated from the French by CHARLES MARTEL; and illustrated with Diagrams, &c. Crown 8vo. price 12s. 6d.

Clinton.—The Autobiography and Literary Journal of the late Henry Fynes Clinton, Esq., M.A., Author of the *Fasti Hellenici*, the *Fasti Romani*, &c. Edited by the Rev. C. J. FYNES CLINTON, M.A., Rector of Cromwell, Notts. [*In the press.*

Conversations on Botany. New Edition, improved; with 22 Plates. Fcp. 8vo. price 7s. 6d.; or with the Plates coloured, 12s.

Conybeare and Howson.—The Life and Epistles of Saint Paul: Comprising a complete Biography of the Apostle, and a Translation of his Epistles inserted in Chronological Order. By the Rev. W. J. CONYBEARE, M.A., late Fellow of Trinity College, Cambridge; and the Rev. J. S. HOWSON, M.A., Principal of the Collegiate Institution, Liverpool. With 40 Engravings on Steel and 100 Woodcuts. 2 vols. 4to. price £2. 8s.

Copland.—A Dictionary of Practical Medicine: Comprising General Pathology, the Nature and Treatment of Diseases, Morbid Structures, and the Disorders especially incidental to Climates, to Sex, and to the different Epochs of Life; with numerous approved Formulae of the Medicines recommended. By JAMES COPLAND, M.D., Consulting Physician to Queen Charlotte's Lying-in Hospital, &c. Vols. I. and II. 8vo. price £3; and Parts X. to XVI. 4s. 6d. each.

The Children's Own Sunday-Book. By JULIA CORNER, Author of *Questions on the History of Europe*. With Two Illustrations. Square fcp. 8vo. price 5s.

Cresy.—An Encyclopædia of Civil Engineering, Historical, Theoretical, and Practical. By EDWARD CRESY, F.S.A., C.E. Illustrated by upwards of 3,000 Woodcuts, explanatory of the Principles, Machinery, and Constructions which come under the direction of the Civil Engineer. 8vo. price £3. 13s. 6d.

The Cricket-Field; or, the Science and History of the Game of Cricket. By the Author of *Principles of Scientific Batting*. Second Edition, greatly improved; with Plates and Woodcuts. Fcp. 8vo. price 5s. half-bound.

Lady Cust's Invalid's Book.—The Invalid's Own Book: A Collection of Recipes from various Books and various Countries. By the Honourable LADY CUST. Fcp. 8vo. price 3s. 6d.

Dale.—The Domestic Liturgy and Family Chaplain, in Two Parts: The First Part being Church Services adapted for Domestic Use, with Prayers for every day of the week, selected exclusively from the Book of Common Prayer; Part II. comprising an appropriate Sermon for every Sunday in the year. By the Rev. THOMAS DALE, M.A., Canon Residentiary of St. Paul's. Second Edition. Post 4to. price 21s. cloth; 31s. 6d. calf; or £2. 10s. morocco.

Separately { THE FAMILY CHAPLAIN, 12s. / THE DOMESTIC LITURGY, 10s. 6d.

Delabeche.—The Geological Observer. By Sir HENRY T. DELABECHE, F.R.S., Director-General of the Geological Survey of the United Kingdom. New Edition; with numerous Woodcuts. 8vo. price 18s.

Delabeche.—Report on the Geology of Cornwall, Devon, and West Somerset. By Sir HENRY T. DELABECHE, F.R.S., Director-General of the Geological Survey. With Maps, Woodcuts, and 12 Plates. 8vo. price 14s.

De la Rive.—A Treatise on Electricity, in Theory and Practice. By A. DE LA RIVE, Professor in the Academy of Geneva. In Two Volumes, with numerous Wood Engravings. Vol. I. 8vo. price 18s.

Discipline. By the Author of "Letters to my Unknown Friends," &c. Second Edition, enlarged. 18mo. price 2s. 6d.

Eastlake.—Materials for a History of Oil Painting. By Sir CHARLES LOCK EASTLAKE, F.R.S., F.S.A., President of the Royal Academy. 8vo. price 16s.

The Eclipse of Faith; or, a Visit to a Religious Sceptic. Fifth *and cheaper* Edition. Fcp. 8vo. price 5s.

A Defence of The Eclipse of Faith, by its Author: Being a Rejoinder to Professor Newman's *Reply*: Including a full Examination of that Writer's Criticism on the Character of Christ; and a Chapter on the Aspects and Pretensions of Modern Deism. *Second Edition*, revised. Post 8vo. 5s. 6d.

The Englishman's Greek Concordance of the New Testament: Being an Attempt at a Verbal Connexion between the Greek and the English Texts; including a Concordance to the Proper Names, with Indexes, Greek-English and English-Greek. New Edition, with a new Index. Royal 8vo. price 42s.

The Englishman's Hebrew and Chaldee Concordance of the Old Testament: Being an Attempt at a Verbal Connection between the Original and the English Translations; with Indexes, a List of the Proper Names and their occurrences, &c. 2 vols. royal 8vo. £3. 13s. 6d.; large paper, £4. 14s. 6d.

Ephemera.—A Handbook of Angling; Teaching Fly-fishing, Trolling, Bottom-fishing, Salmon fishing; with the Natural History of River Fish, and the best modes of Catching them. By EPHEMERA. Third *and cheaper* Edition, corrected and improved; with Woodcuts. Fcp. 8vo. 5s.

Ephemera.—The Book of the Salmon: Comprising the Theory, Principles, and Practice of Fly-fishing for Salmon; Lists of good Salmon Flies for every good River in the Empire; the Natural History of the Salmon, all its known Habits described, and the best way of artificially Breeding it explained. With numerous coloured Engravings. By EPHEMERA; assisted by ANDREW YOUNG. Fcp. 8vo. with coloured Plates, price 14s.

W. Erskine, Esq.—History of India under Báber and Humáyun, the First Two Sovereigns of the House of Taimur. By WILLIAM ERSKINE, Esq., Editor of *Memoirs of the Emperor Báber*. 8vo. price 32s.

Faraday (Professor).—The Subject-Matter of Six Lectures on the Non-Metallic Elements, delivered before the Members of the Royal Institution in 1852, by Professor FARADAY, D.C.L., F.R.S., &c. Arranged by permission from the Lecturer's Notes by J. SCOFFERN, M.B. Fcp. 8vo. price 5s. 6d.

Norway in 1848 and 1849: Containing Rambles among the Fjelds and Fjords of the Central and Western Districts; and including Remarks on its Political, Military, Ecclesiastical, and Social Organisation. By THOMAS FORESTER, Esq.; and Lieutenant M. S. BIDDULPH, Royal Artillery. With Map, Woodcuts, and Plates. 8vo. 18s.

Francis.—Annals, Anecdotes, and Legends: A Chronicle of Life Assurance. By JOHN FRANCIS, Author of *The History of the Bank of England*, &c. Post 8vo. 8s. 6d.

Fullom.—The Marvels of Science and their Testimony to Holy Writ: A Popular System of the Sciences. By S. W. FULLOM, Esq. The *Eighth* and cheaper *Edition*; with numerous Illustrations. Post 8vo. price 5s.

The Poetical Works of Oliver Goldsmith. Edited by BOLTON CORNEY, Esq. Illustrated by Wood Engravings, from Designs by Members of the Etching Club. Square crown 8vo. cloth, 21s.; morocco, £1. 16s.

Gosse. — A Naturalist's Sojourn in Jamaica. By P. H. GOSSE, Esq. With Plates. Post 8vo. price 14s.

Mr. W. R. Greg's Contributions to The Edinburgh Review.—Essays on Political and Social Science. Contributed chiefly to the *Edinburgh Review*. By WILLIAM R. GREG. 2 vols. 8vo. price 24s.

Gurney.—Historical Sketches; illustrat-ing some Memorable Events and Epochs, from A.D. 1,400 to A.D. 1,546. By the Rev. JOHN HAMPDEN GURNEY, M.A., Rector of St. Mary's, Marylebone. Fcp. 8vo. 7s. 6d.

Gwilt.—An Encyclopædia of Architecture, Historical, Theoretical, and Practical. By JOSEPH GWILT. Illustrated with more than One Thousand Engravings on Wood, from Designs by J. S. GWILT. Third Edition (1854). 8vo. price 42s.

Sidney Hall's General Large Library Atlas of Fifty-three Maps (size, 20 in. by 16 in.), with the Divisions and Boundaries carefully coloured; and an Alphabetical Index of all the Names contained in the Maps. New Edition, corrected from the best and most recent Authorities; with the Railways laid down and many entirely new Maps. Colombier 4to. price £5. 5s. half-russia.

Hamilton. — Discussions in Philosophy and Literature, Education and University Reform. Chiefly from the *Edinburgh Review*; corrected, vindicated, enlarged. in Notes and Appendices. By Sir WILLIAM HAMILTON, Bart. Second Edition, with Additions. 8vo. price 21s.

Hare (Archdeacon).—The Life of Luther, in Forty-eight Historical Engravings. By GUSTAV KÖNIG. With Explanations by Archdeacon HARE. Square crown 8vo. [*In the press.*

Harrison.—The Light of the Forge; or, Counsels drawn from the Sick-Bed of E. M. By the Rev. WILLIAM HARRISON, M.A., Rector of Birch, Essex, and Domestic Chaplain to H.R.H. the Duchess of Cambridge. With 2 Woodcuts. Fcp. 8vo. price 5s.

Harry Hieover. — The Hunting-Field. By HARRY HIEOVER. With Two Plates—One representing *The Right Sort*; the other, *The Wrong Sort*. Fcp. 8vo. 5s. half-bound.

Harry Hieover. — Practical Horseman-ship. By HARRY HIEOVER. With 2 Plates—One representing *Going like Workmen*; the other, *Going like Muffs*. Fcp. 8vo. price 5s. half-bound.

Harry Hieover.—The Stud, for Practical Purposes and Practical Men: being a Guide to the Choice of a Horse for use more than for show. By HARRY HIEOVER. With 2 Plates—One representing *A pretty good sort for most purposes*; the other, *Rayther a bad sort for any purpose*. Fcp. 8vo. price 5s. half-bound.

Harry Hieover. — The Pocket and the Stud; or, Practical Hints on the Management of the Stable. By HARRY HIEOVER. Second Edition; with Portrait of the Author on his favourite Horse *Harlequin*. Fcp. 8vo. price 5s. half-bound.

Harry Hieover.—Stable Talk and Table Talk; or, Spectacles for Young Sportsmen. By HARRY HIEOVER. New Edition, 2 vols. 8vo. with Portrait, price 24s.

Haydon.—The Life of Benjamin Robert Haydon, Historical Painter, from his Autobiography and Journals. Edited and compiled by TOM TAYLOR, M.A., of the Inner Temple, Esq.; late Fellow of Trinity College, Cambridge; and late Professor of the English Language and Literature in University College, London. Second Edition, with Additions and an Index. 3 vols. post 8vo. price 31s. 6d.

Haydn's Book of Dignities: Containing Rolls of the Official Personages of the British Empire, Civil, Ecclesiastical, Judicial, Military, Naval, and Municipal, from the Earliest Periods to the Present Time; Compiled chiefly from the Records of the Public Offices. Together with the Sovereigns of Europe, from the foundation of their respective States; the Peerage and Nobility of Great Britain, and numerous other Lists. Being a New Edition, improved and continued, of Beatson's Political Index. By JOSEPH HAYDN, Compiler of *The Dictionary of Dates*, and other Works. 8vo. price 25s. half-bound.

Sir John Herschel.—Outlines of Astronomy. By Sir John F. W. Herschel, Bart. &c. New Edition; with Plates and Wood Engravings. 8vo. price 18s.

Hill.—Travels in Siberia. By S. S. Hill, Esq. Author of *Travels on the Shores of the Baltic.* With a large coloured Map of European and Asiatic Russia. 2 vols. post 8vo. price 24s.

Hints on Etiquette and the Usages of Society: With a Glance at Bad Habits. New Edition, revised (with Additions) by a Lady of Rank. Fcp. 8vo. price Half-a-Crown.

Hole.—Prize Essay on the History and Management of Literary, Scientific, and Mechanics' Institutions, and especially how far they may be developed and combined so as to promote the Moral Well-being and Industry of the Country. By James Hole, Hon. Secretary of the Yorkshire Union of Mechanics' Institutes. 8vo. price 5s.

Lord Holland's Memoirs.—Memoirs of the Whig Party during my Time. By Henry Richard Lord Holland. Edited by his Son, Henry Edward Lord Holland. Vols. I. and II. post 8vo. price 9s. 6d. each.

Lord Holland's Foreign Reminiscences. Edited by his Son, Henry Edward Lord Holland. Second Edition; with Facsimile. Post 8vo. price 10s. 6d.

Holland.—Chapters on Mental Physiology. By Sir Henry Holland, Bart., F.R.S., Physician-Extraordinary to the Queen; and Physician in Ordinary to His Royal Highness Prince Albert. Founded chiefly on Chapters contained in *Medical Notes and Reflections* by the same Author. 8vo. price 10s. 6d.

Hook.—The Last Days of Our Lord's Ministry: A Course of Lectures on the principal Events of Passion Week. By Walter Farquhar Hook, D.D., Chaplain in Ordinary to the Queen. New Edition. Fcp. 8vo. price 6s.

Hooker and Arnott.—The British Flora; Comprising the Phaenogamous or Flowering Plants, and the Ferns. The Sixth Edition, with Additions and Corrections; and numerous Figures illustrative of the Umbelliferous Plants, the Composite Plants, the Grasses, and the Ferns. By Sir W. J. Hooker, F.R.A. and L.S., &c., and G. A. Walker-Arnott, LL.D., F.L.S. 12mo. with 12 Plates, price 14s.; with the Plates coloured, price 21s.

Hooker.—Kew Gardens; or, a Popular Guide to the Royal Botanic Gardens of Kew. By Sir William Jackson Hooker, K.H., D.C.L., F.R.A., and L.S., &c. &c. Director. New Edition; with numerous Wood Engravings. 16mo. price Sixpence.

Horne.—An Introduction to the Critical Study and Knowledge of the Holy Scriptures. By Thomas Hartwell Horne, B.D. of St. John's College, Cambridge; Prebendary of St. Paul's. New Edition, revised and corrected; with numerous Maps and Facsimiles of Biblical Manuscripts. 5 vols 8vo. price 63s.

Horne.—A Compendious Introduction to the Study of the Bible. By Thomas Hartwell Horne, B.D., of St. John's College, Cambridge. Being an Analysis of his *Introduction to the Critical Study and Knowledge of the Holy Scriptures.* New Edition, corrected and enlarged; with Maps and other Engravings. 12mo. price 9s.

Howitt (A. M.)—An Art-Student in Munich. By Anna Mary Howitt. 2 vols. post 8vo. price 14s.

Howitt.—The Children's Year. By Mary Howitt. With Four Illustrations, engraved by John Absolon, from Original Designs by Anna Mary Howitt. Square 16mo. price 5s.

William Howitt's Boy's Country Book; Being the Real Life of a Country Boy, written by himself; exhibiting all the Amusements, Pleasures, and Pursuits of Children in the Country. New Edition; with 40 Woodcuts. Fcp. 8vo. price 6s.

Howitt.—The Rural Life of England. By William Howitt. New Edition, corrected and revised; with Woodcuts by Bewick and Williams: Uniform with *Visits to Remarkable Places.* Medium 8vo. 21s.

Howitt.—Visits to Remarkable Places; Old Halls, Battle-Fields, and Scenes illustrative of Striking Passages in English History and Poetry. By William Howitt. New Edition, with 40 Woodcuts. Medium 8vo. price 21s.

Second Series, chiefly in the Counties of Northumberland and Durham, with a Stroll along the Border. With upwards of 40 Woodcuts. Medium 8vo. 21s.

Hudson.—Plain Directions for Making Wills in Conformity with the Law: with a clear Exposition of the Law relating to the distribution of Personal Estate in the case of Intestacy, two Forms of Wills, and much useful information. By J. C. HUDSON, Esq., late of the Legacy Duty Office, London. New and enlarged Edition; including the provisions of the Wills Act Amendment Act of 1852 (introduced by Lord St. Leonard's). Fcp. 8vo. price 2s. 6d.

Hudson. — The Executor's Guide. By J. C. HUDSON, Esq. New and enlarged Edition; with the Addition of Directions for paying Succession Duties on Real Property under Wills and Intestacies, and a Table for finding the Values of Annuities and the Amount of Legacy and Succession Duty thereon. Fcp. 8vo. price 6s.

Hulbert.—The Gospel Revealed to Job; or, Patriarchal Faith illustrated in Thirty Lectures on the principal Passages of the Book of Job: With Explanatory, Illustrative, and Critical Notes. By the Rev. C. A. HULBERT, M.A. 8vo. price 12s.

Humbley.—Journal of a Cavalry Officer: Including the memorable Sikh Campaign of 1845-6. By W. W. W. HUMBLEY, M.A. Trinity College, Cambridge; Fellow of the Cambridge Philosophical Society; Captain, 9th Queen's Royal Lancers. With Plans and Map. Royal 8vo. price 21s.

Humboldt's Aspects of Nature. Trans-lated, with the Author's authority, by Mrs. SABINE. New Edition. 16mo. price 6s.: or in 2 vols. 3s. 6d. each, cloth; 2s. 6d. each, sewed.

Humboldt's Cosmos. Translated, with the Author's authority, by Mrs. SABINE. Vols. I. and II. 16mo. Half-a-Crown each, sewed; 3s. 6d. each, cloth: or in post 8vo. 12s. 6d. each, cloth. Vol. III. post 8vo. 12s. 6d. cloth: or in 16mo. Part I. 2s. 6d. sewed, 3s. 6d. cloth; and Part II. 3s. sewed, 4s. cloth.

Humphreys.—Sentiments and Similes of Shakspeare: A Classified Selection of Similes, Definitions, Descriptions, and other remarkable Passages in Shakspeare's Plays and Poems. With an elaborately illuminated border in the characteristic style of the Elizabethan Period, massive carved covers, and other Embellishments, designed and executed by H. N. HUMPHREYS. Square post 8vo. price 21s.

Hunt. — Researches on Light in its Chemical Relations; embracing a Consideration of all the Photographic Processes. By ROBERT HUNT, F.R.S., Professor of Physics in the Metropolitan School of Science. Second Edition, thoroughly revised; with extensive Additions, a Plate, and Woodcuts. 8vo. price 10s. 6d.

Jameson. — A Commonplace Book of Thoughts, Memories, and Fancies, Original and Selected. Part I. Ethics and Character; Part II. Literature and Art. By Mrs. JAMESON. With Etchings and Wood Engravings. Square crown 8vo. [*Just ready.*

Mrs. Jameson's Legends of the Saints and Martyrs. Forming the First Series of *Sacred and Legendary Art.* Second Edition; with numerous Woodcuts, and 16 Etchings by the Author. Square crown 8vo. price 28s.

Mrs. Jameson's Legends of the Monastic Orders, as represented in the Fine Arts. Forming the Second Series of *Sacred and Legendary Art.* Second Edition, corrected and enlarged; with 11 Etchings by the Author, and 88 Woodcuts. Square crown 8vo. price 28s.

Mrs. Jameson's Legends of the Madonna, as represented in the Fine Arts. Forming the Third Series of *Sacred and Legendary Art.* With 55 Drawings by the Author, and 152 Wood Engravings. Square crown 8vo. price 28s.

Lord Jeffrey's Contributions to The Edinburgh Review. A New Edition, complete in One Volume, with a Portrait engraved by Henry Robinson, and a Vignette View of Craigcrook engraved by J. Cousen. Square crown 8vo. 21s. cloth; or 30s. calf.

*** Also a LIBRARY EDITION, in 3 vols. 8vo. price 42s.

Bishop Jeremy Taylor's Entire Works: With Life by Bishop HEBER. Revised and corrected by the Rev. CHARLES PAGE EDEN, Fellow of Oriel College, Oxford. Now complete in Ten Volumes 8vo price Half-a-Guinea each.

Jesse. — Russia and the War. By Captain JESSE (late Unattached), Author of Murray's *Handbook for Russia*, &c. With a Plan of the Town and Harbour of Sebastopool, shewing the Batteries and Approaches. Crown 8vo. price 2s. 6d.

Johnston.—A New Dictionary of Geography, Descriptive, Physical, Statistical, and Historical: Forming a complete General Gazetteer of the World. By ALEXANDER KEITH JOHNSTON, F.R.S.E., F.R.G.S., F.G.S., Geographer at Edinburgh in Ordinary to Her Majesty. In One Volume of 1,440 pages; comprising nearly 50,000 Names of Places. 8vo. price 36s. cloth; or half-bound in russia, 41s.

Kemble.—The Saxons in England: A History of the English Commonwealth till the period of the Norman Conquest. By JOHN MITCHELL KEMBLE, M.A., F.C.P.S., &c. 2 vols. 8vo. price 28s.

Kent. — Aletheia; or, the Doom of Mythology. With other Poems. By WILLIAM CHARLES MARK KENT. Fcp. 8vo. price 7s. 6d.

Kippis's Collection of Hymns and Psalms for Public and Private Worship. New Edition; including a New Supplement by the Rev. EDMUND KELL, M.A. 18mo. price 4s. cloth; or 4s. 6d. roan.

Kirby and Spence's Introduction to Entomology; or, Elements of the Natural History of Insects: Comprising an account of noxious and useful Insects, of their Metamorphoses, Food, Stratagems, Habitations, Societies, Motions, Noises, Hybernation, Instinct, &c. New Edition. 2 vols. 8vo. with Plates, price 31s. 6d.

Kirby.—The Life of the Rev. William Kirby, M.A., Rector of Barham; Author of one of the Bridgewater Treatises, and Joint-Author of the *Introduction to Entomology*. By the Rev. JOHN FREEMAN, M.A., With Portrait, Vignette, and Facsimile. 8vo. price 15s.

The Lady Una and her Queendom; or, Reform at the Right End. By the Author of *Home Truths for Home Peace*, &c. Fcp. 8 vo. price 7s.

Laing's (S.) Observations on the Social and Political State of Denmark and the Duchies of Sleswick and Holstein in 1851: Being the Third Series of *Notes of a Traveller*. 8vo. price 12s.

Laing's (S.) Observations on the Social and Political State of the European People in 1848 and 1849: Being the Second Series of *Notes of a Traveller*. 8vo. price 14s.

*** The *First Series*, in 16mo. price 2s. 6d.

Dr. Latham on Diseases of the Heart. Lectures on Subjects connected with Clinical Medicine: Diseases of the Heart. By P. M. LATHAM, M.D., Physician Extraordinary to the Queen. New Edition. 2 vols. 12mo. price 16s.

Mrs. R. Lee's Elements of Natural History; or, First Principles of Zoology: Comprising the Principles of Classification, interspersed with amusing and instructive Accounts of the most remarkable Animals. New Edition, enlarged, with numerous additional Woodcuts. Fcp. 8vo. price 7s. 6d.

L. E. L.—The Poetical Works of Letitia Elizabeth Landon; comprising the *Improvisatrice*, the *Venetian Bracelet*, the *Golden Violet*, the *Troubadour*, and Poetical Remains. New Edition; with 2 Vignettes by R. Doyle. 2 vols. 16mo. 10s. cloth; morocco, 21s.

Letters on Happiness, addressed to a Friend. By the Author of *Letters to My Unknown Friends*, &c. Fcp. 8vo. price 6s.

Letters to my Unknown Friends. By a LADY, Author of *Letters on Happiness*. Fourth *and cheaper* Edition. Fcp. 8vo. price 5s.

Lindley.—The Theory of Horticulture; Or, an Attempt to explain the principal Operations of Gardening upon Physiological Principles. By John Lindley, Ph.D. F.R.S. New Edition, revised and improved; with Wood Engravings. 8vo. [*In the press.*

Dr. John Lindley's Introduction to Botany. New Edition, with Corrections and copious Additions. 2 vols. 8vo. with Six Plates and numerous Woodcuts, price 24s.

Linwood.—Anthologia Oxoniensis, sive Florilegium e lusibus poeticis diversorum Oxoniensium Græcis et Latinis decerptum. Curante GULIELMO LINWOOD, M.A. Ædis Christi Alummo. 8vo. price 14s.

Dr. Little on Deformities.—On the Nature and Treatment of Deformities of the Human Frame. By W. J. LITTLE, M.D., Physician to the London Hospital, Founder of the Royal Orthopædic Hospital, &c. With 160 Woodcuts and Diagrams. 8vo. price 15s.

LARDNER'S CABINET CYCLOPÆDIA

Of History, Biography, Literature, the Arts and Sciences, Natural History, and Manufactures: A Series of Original Works by

SIR JOHN HERSCHEL,	THOMAS KEIGHTLEY,	BISHOP THIRLWALL,
SIR JAMES MACKINTOSH,	JOHN FORSTER,	THE REV. G. R. GLEIG,
ROBERT SOUTHEY,	SIR WALTER SCOTT,	J. C. L. DE SISMONDI,
SIR DAVID BREWSTER,	THOMAS MOORE,	JOHN PHILLIPS, F.R.S. G.S.

AND OTHER EMINENT WRITERS.

Complete in 132 vols. fcp. 8vo. with Vignette Titles, price, in cloth, Nineteen Guineas.
The Works *separately*, in Sets or Series, price Three Shillings and Sixpence each Volume.

A List of the WORKS *composing the* CABINET CYCLOPÆDIA:—

1. Bell's History of Russia......3 vols. 10s. 6d.
2. Bell's Lives of British Poets..2 vols. 7s.
3. Brewster's Optics.......... .1 vol. 3s. 6d.
4. Cooley's Maritime and Inland Discovery ....3 vols. 10s. 6d
5. Crowe's History of France....3 vols. 10s. 6d.
6. De Morgan on Probabilities ..1 vol. 3s. 6d.
7. De Sismondi's History of the Italian Republics..........1 vol. 3s. 6d.
8. De Sismondi's Fall of the Roman Empire............2 vols. 7s.
9. Donovan's Chemistry ....1 vol. 3s. 6d.
10. Donovan's Domestic Economy,2 vols. 7s.
11. Dunham's Spain and Portugal,5 vols. 17s. 6d.
12. Dunham's History of Denmark, Sweden, and Norway ......3 vols. 10s. 6d.
13. Dunham's History of Poland..1 vol. 3s. 6d.
14. Dunham's Germanic Empire..3 vols. 10s. 6d.
15. Dunham's Europe during the Middle Ages ....4 vols. 14s.
16. Dunham's British Dramatists, 2 vols. 7s.
17. Dunham's Lives of Early Writers of Great Britain ..1 vol. 3s. 6d.
18. Fergus's History of the United States .... 2 vols. 7s.
19. Fosbroke's Grecian and Roman Antiquities ....2 vols. 7s.
20. Forster's Lives of the Statesmen of the Commonwealth, 5 vols. 17s. 6d.
21. Gleig's Lives of British Military Commanders..........3 vols. 10s. 6d.
22. Grattan's History of the Netherlands ....1 vol. 3s. 6d.
23. Henslow's Botany............1 vol. 3s. 6d.
24. Herschel's Astronomy........1 vol. 3s. 6d.
25. Herschel's Discourse on Natural Philosophy ....1 vol. 3s. 6d.
26. History of Rome.............2 vols. 7s.
27. History of Switzerland ....1 vol. 3s. 6d.
28. Holland's Manufactures in Metal ....3 vols. 10s. 6d.
29. James's Lives of Foreign Statesmen ....5 vols. 17s. 6d.
30. Kater and Lardner's Mechanics,1 vol. 3s. 6d.
31. Keightley's Outlines of History,1 vol. 3s. 6d.
32. Lardner's Arithmetic ....1 vol. 3s. 6d.
33. Lardner's Geometry ....1 vol. 3s. 6d.
34. Lardner on Heat ....1 vol. 3s. 6d.
35. Lardner's Hydrostatics and Pneumatics ....1 vol. 3s. 6d.
36. Lardner and Walker's Electricity and Magnetism ......2 vols. 7s.
37. Mackintosh, Forster, and Courtenay's Lives of British Statesmen .... 7 vols. 24s. 6d.
38. Mackintosh, Wallace, and Bell's History of England........10 vols. 35s.
39. Montgomery and Shelley's eminent Italian, Spanish, and Portuguese Authors . 3 vols. 10s. 6d.
40. Moore's History of Ireland ..4 vols. 14s.
41. Nicolas's Chronology of Hist. 1 vol. 3s. 6d.
42. Phillips's Treatise on Geology, 2 vols. 7s.
43. Powell's History of Natural Philosophy...............1 vol. 3s. 6d.
44. Porter's Treatise on the Manufacture of Silk ....1 vol. 3s. 6d.
45. Porter's Manufactures of Porcelain and Glass ....1 vol. 3s. 6d.
46. Roscoe's British Lawyers ....1 vol. 3s 6d.
47. Scott's History of Scotland ....2 vols. 7s.
48. Shelley's Lives of eminent French Authors............2 vols. 7s.
49. Shuckard and Swainson's Insects, 1 vol. 3s. 6d.
50. Southey's Lives of British Admirals...................5 vols. 17s. 6d.
51. Stebbing's Church History....2 vols. 7s.
52. Stebbing's History of the Reformation ....2 vols. 7s.
53. Swainson's Discourse on Natural History..............1 vol. 3s 6d.
54. Swainson's Natural History & Classification of Animals ..1 vol. 3s. 6d.
55. Swainson's Habits & Instincts of Animals ....1 vol. 3s. 6d.
56. Swainson's Birds ....2 vols. 7s.
57. Swainson's Fish, Reptiles, &c. 2 vols. 7s.
58. Swainson's Quadrupeds ......1 vol. 3s. 6d.
59. Swainson's Shells and Shell-fish, 1 vol. 3s. 6d.
60. Swainson's Animals in Menageries ....1 vol. 3s. 6d.
61. Swainson's Taxidermy and Biography of Zoologists....1 vol. 3s. 6d.
62. Thirlwall's History of Greece..8 vols. 28s.

Litton.—The Church of Christ, in its Idea, Attributes, and Ministry: With a particular Reference to the Controversy on the Subject between Romanists and Protestants. By the Rev. EDWARD ARTHUR LITTON, M.A., Vice-Principal of St. Edmund Hall, Oxford. 8vo. price 16s.

Loch.—A Practical Legal Guide for Sailors and Merchants during War: Comprising Blockade, Captors, Cartel, Colours, Contraband, Droits of Admiralty, Flag Share, Freight, Head Money, Joint Capture, Neutrals and Neutral Territory, Prizes, Recapture of Property of Ally, Rescue, Right of Visit and Search, Salvage, Derelict, Trading with the Enemy, Orders in Council, &c., Prize Act, Proclamation as to Colours. With Appendices containing the Orders in Council and other Official Documents relating to the Present War. By WILLIAM ADAM LOCH, of the Hon. Society of Lincoln's Inn. 8vo. price 9s. 6d.

Lorimer's (C.) Letters to a Young Master Mariner on some Subjects connected with his calling. New Edition. Fcp. 8vo. 5s. 6d.

Loudon's Self-Instruction for Young Gardeners, Foresters, Bailiffs, Land Stewards, and Farmers; in Arithmetic, Book-keeping, Geometry, Mensuration, Practical Trigonometry, Mechanics, Land-Surveying, Levelling, Planning and Mapping, Architectural Drawing, and Isometrical Projection and Perspective: With Examples shewing their applications to Horticulture and Agricultural Purposes; a Memoir, Portrait, and Woodcuts. 8vo. price 7s. 6d.

Loudon's Encyclopædia of Gardening; comprising the Theory and Practice of Horticulture, Floriculture, Arboriculture, and Landscape Gardening: Including all the latest improvements; a General History of Gardening in all Countries; a Statistical View of its Present State; and Suggestions for its Future Progress in the British Isles. With many hundred Woodcuts. New Edition, corrected and improved by Mrs. LOUDON. 8vo. price 50s.

Loudon's Encyclopædia of Trees and Shrubs; or, the *Arboretum et Fruticetum Britannicum* abridged: Containing the Hardy Trees and Shrubs of Great Britain, Native and Foreign, Scientifically and Popularly Described; with their Propagation, Culture, and Uses in the Arts; and with Engravings of nearly all the Species. Adapted for the use of Nurserymen, Gardeners, and Foresters. With about 2,000 Woodcuts. 8vo. price 50s.

Loudon's Encyclopædia of Agriculture; comprising the Theory and Practice of the Valuation, Transfer, Laying-out, Improvement, and Management of Landed Property, and of the Cultivation and Economy of the Animal and Vegetable Productions of Agriculture; Including all the latest Improvements, a general History of Agriculture in all Countries, a Statistical View of its present State, and Suggestions for its future progress in the British Isles. New Edition; with 1,100 Woodcuts. 8vo. price 50s.

Loudon's Encyclopædia of Plants, in- cluding all which are now found in, or have been introduced into, Great Britain: Giving their Natural History, accompanied by such descriptions, engraved figures, and elementary details, as may enable a beginner, who is a mere English reader, to discover the name of every Plant which he may find in flower, and acquire all the information respecting it which is useful and interesting. New Edition, corrected throughout and brought down to the year 1855, by Mrs. LOUDON and GEORGE DON, Esq., F.L.S. &c., 8vo. [*In the Spring.*

Loudon's Encyclopædia of Cottage, Farm, and Villa Architecture and Furniture: containing numerous Designs, from the Villa to the Cottage and the Farm, including Farm Houses, Farmeries, and other Agricultural Buildings; Country Inns, Public Houses, and Parochial Schools; with the requisite Fittings-up, Fixtures, and Furniture, and appropriate Offices, Gardens, and Garden Scenery: Each Design accompanied by Analytical and Critical Remarks. New Edition, edited by Mrs. LOUDON; with more than 2,000 Woodcuts. 8vo. price 63s.

Loudon's Hortus Britannicus; or, Cata- logue of all the Plants indigenous to, cultivated in, or introduced into Britain. An entirely New Edition, corrected throughout; With a Supplement, including all the New Plants, and a New General Index to the whole Work. Edited by Mrs. LOUDON; assisted by W. H. BAXTER and DAVID WOOSTER. 8vo. price 31s. 6d.—The SUPPLEMENT separately, price 14s.

Mrs. Loudon's Amateur Gardener's Calendar: Being a Monthly Guide as to what should be avoided as well as what should be done, in a Garden in each Month; with plain Rules *how to do* what is requisite; Directions for Laying Out and Planting Kitchen and Flower Gardens, Pleasure Grounds, and Shrubberies: and a short Account, in each Month, of the Quadrupeds, Birds, and Insects then most injurious to Gardens. 16mo. with Woodcuts, price 7s. 6d.

Mrs. Loudon's Lady's Country Companion; or, How to enjoy a Country Life Rationally. Fourth Edition; with Plates and Wood Engravings. Fcp. 8vo. price 5s.

Low.—A Treatise on the Domesticated Animals of the British Islands: Comprehending the Natural and Economical History of Species and Varieties; the Description of the Properties of external Form; and Observations on the Principles and Practice of Breeding. By D. Low, Esq., F.R.S.E. With Wood Engravings. 8vo. price 25s.

Low.—Elements of Practical Agriculture; comprehending the Cultivation of Plants, the Husbandry of the Domestic Animals, and the Economy of the Farm. By D. Low, Esq. F.R.S.E. New Edition; with 200 Woodcuts. 8vo. price 21s.

Macaulay.—Speeches of the Right Hon. T. B. Macaulay, M.P. Corrected by Himself. 8vo. price 12s.

Macaulay.—The History of England from the Accession of James II. By Thomas Babington Macaulay. New Edition. Vols. I. and II. 8vo. price 32s.

Mr. Macaulay's Critical and Historical Essays contributed to The Edinburgh Review. Four Editions, as follows:—

1. Library Edition (the *Seventh*), in 3 vols. 8vo. price 36s.
2. Complete in One Volume, with Portrait and Vignette. Square crown 8vo. price 21s. cloth; or 30s. calf.
3. A New Edition, in 3 vols. fcp. 8vo. price 21s.
4. People's Edition, in 2 vols. crown 8vo. price 8s. cloth.

Macaulay.—Lays of Ancient Rome, with Ivry and the Armada. By Thomas Babington Macaulay. New Edition. 16mo. price 4s. 6d. cloth; or 10s. 6d. bound in morocco.

Mr. Macaulay's Lays of Ancient Rome. With numerous Illustrations, Original and from the Antique, drawn on Wood by George Scarf, Jun., and engraved by Samuel Williams. New Edition. Fcp. 4to. price 21s. boards; or 42s. bound in morocco.

Macdonald.—Villa Verocchio; or, the Youth of Leonardo da Vinci: A Tale. By the late Diana Louisa Macdonald. Fcp. 8vo. price 6s.

Macintosh.—A Military Tour in European Turkey, the Crimea, and on the Eastern Shores of the Black Sea: Including Routes across the Balkan into Bulgaria, and Excursions in the Turkish, Russian, and Persian Provinces of the Caucasian Range; with Strategical Observations on the Probable Scene of the Operations of the Allied Expeditionary Force. By Major-Gen. A. F. Macintosh, K.H., F.R.G.S., F.G.S., Commanding Her Majesty's Troops in the Ionian Islands. With Maps. 2 vols. post 8vo. price 21s.

Sir James Mackintosh's History of England from the Earliest Times to the final Establishment of the Reformation. Library Edition, revised by the Author's Son. 2 vols. 8vo. price 21s.

Sir James Mackintosh's Miscellaneous Works: Including his Contributions to The Edinburgh Review. Complete in One Volume; with Portrait and Vignette. Square crown 8vo. price 21s. cloth; or 30s. bound in calf.

*** Also a NEW EDITION, in 3 vols. fcp. 8vo. price 21s.

M'Culloch.—A Dictionary, Practical, Theoretical, and Historical, of Commerce and Commercial Navigation. Illustrated with Maps and Plans. By J. R. M'Culloch, Esq. New Edition; and embracing a large mass of new and important Information in regard to the Trade, Commercial Law, and Navigation of this and other Countries. 8vo. price 50s. cloth; half-russia, 55s.

M'Culloch.—A Dictionary, Geographical, Statistical, and Historical, of the various Countries, Places, and principal Natural Objects in the World. By J. R. M'Culloch, Esq. Illustrated with Six large Maps. New Edition; with a Supplement, comprising the Population of Great Britain from the Census of 1851. 2 vols. 8vo. price 63s.

M'Culloch.—An Account, Descriptive and Statistical, of the British Empire; Exhibiting its Extent, Physical Capacities, Population, Industry, and Civil and Religious Institutions. By J. R. M'Culloch, Esq. Fourth Edition (1854), revised; with an Appendix of Tables. 2 vols. 8vo. price 42s.

Maitland.—The Church in the Catacombs: A Description of the Primitive Church of Rome. Illustrated by its Sepulchral Remains. By the Rev. Charles Maitland. New Edition; with many Woodcuts. 8vo. price 14s.

Mrs. Marcet's Conversations on Chemistry, in which the Elements of that Science are familiarly explained and illustrated by Experiments. New Edition, enlarged and improved. 2 vols. fcp. 8vo. price 14s.

Mrs. Marcet's Conversations on Natural Philosophy, in which the Elements of that Science are familiarly explained. New Edition, enlarged and corrected; with 23 Plates. Fcp. 8vo. price 10s. 6d.

Mrs. Marcet's Conversations on Political Economy, in which the Elements of that Science are familiarly explained. New Edition. Fcp. 8vo. price 7s. 6d.

Mrs. Marcet's Conversations on Vegetable Physiology; comprehending the Elements of Botany, with their Application to Agriculture. New Edition; with 4 Plates. Fcp. 8vo. price 9s.

Mrs. Marcet's Conversations on Land and Water. New Edition, revised and corrected; with a coloured Map, shewing the comparative Altitude of Mountains. Fcp. 8vo. price 5s. 6d.

Martineau.—Church History in England: Being a Sketch of the History of the Church of England from the Earliest Times to the Period of the Reformation. By the Rev. ARTHUR MARTINEAU, M.A. late Fellow of Trinity College, Cambridge. 12mo. price 6s.

Maunder's Biographical Treasury; consisting of Memoirs, Sketches, and brief Notices of above 12,000 Eminent Persons of All Ages and Nations, from the Earliest Period of History; forming a new and complete Dictionary of Universal Biography. The Eighth Edition, revised throughout, and brought down to the close of the year 1853. Fcp. 8vo. 10s. cloth; bound in roan, 12s.; calf lettered, 12s. 6d.

Maunder's Historical Treasury; comprising a General Introductory Outline of Universal History, Ancient and Modern, and a Series of separate Histories of every principal Nation that exists; their Rise, Progress, and Present Condition, the Moral and Social Character of their respective inhabitants, their Religion, Manners and Customs, &c. &c. New Edition; revised throughout and brought down to the Present Time. Fcp. 8vo. 10s. cloth; roan, 12s.; calf, 12s. 6d.

Maunder's Scientific and Literary Treasury: A new and popular Encyclopædia of Science and the Belles-Lettres; including all Branches of Science, and every subject connected with Literature and Art. New Edition. Fcp. 8vo. price 10s. cloth; bound in roan, 12s.; calf lettered, 12s. 6d.

Maunder's Treasury of Natural History; Or, a Popular Dictionary of Animated Nature: In which the Zoological Characteristics that distinguish the different Classes, Genera, and Species, are combined with a variety of interesting Information illustrative of the Habits, Instincts, and General Economy of the Animal Kingdom. With 900 Woodcuts. New Edition. Fcp. 8vo. price 10s. cloth; roan, 12s.; calf, 12s. 6d.

Maunder's Treasury of Knowledge, and Library of Reference. Comprising an English Dictionary and Grammar, an Universal Gazetteer, a Classical Dictionary, a Chronology, a Law Dictionary, a Synopsis of the Peerage, numerous useful Tables, &c. The Twentieth Edition, carefully revised and corrected throughout: With some Additions. Fcp. 8vo. price 10s. cloth; bound in roan, 12s.; calf lettered, 12s. 6d.

Merivale.—A History of the Romans under the Empire. By the Rev. CHARLES MERIVALE, B.D., late Fellow of St. John's College, Cambridge. Vols. I. and II. 8vo. price 28s.; and Vol. III. price 14s.

Merivale.—The Fall of the Roman Republic: A Short History of the Last Century of the Commonwealth. By the Rev. CHARLES MERIVALE, B.D., late Fellow of St. John's College, Cambridge. 12mo. price 7s. 6d.

Merivale.—An Account of the Life and Letters of Cicero. Translated from the German of Abeken; and edited by the Rev. CHARLES MERIVALE, B.D. 12mo. 9s. 6d.

Milner.—The Baltic; Its Gates, Shores, and Cities: With a Notice of the White Sea, &c. By the Rev. T. MILNER, M.A., F.R.G.S. Post 8vo. [*Just ready.*

Milner's History of the Church of Christ. With Additions by the late Rev. ISAAC MILNER, D.D., F.R.S. A New Edition, revised, with additional Notes by the Rev. T. GRANTHAM, B.D. 4 vols. 8vo. price 52s.

Montgomery.—Memoirs of the Life and Writings of James Montgomery: Including Selections from his Correspondence and Conversations. By JOHN HOLLAND and JAMES EVERETT. [*In the press.*

James Montgomery's Poetical Works: Collective Edition; with the Author's Autobiographical Prefaces, complete in One Volume; with Portrait and Vignette. Square crown 8vo. price 10s. 6d. cloth; morocco, 21s.—Or, in 4 vols. fcp. 8vo. with Portrait, and 7 other Plates, price 20s. cloth; morocco, 36s.

James Montgomery's Original Hymns for Public, Social, and Private Devotion. 18mo. price 5s. 6d.

Moore.—Man and his Motives. By George Moore, M.D., Member of the Royal College of Physicians. *Third* and cheaper *Edition*. Fcp. 8vo. price 6s.

Moore.—The Power of the Soul over the Body, considered in relation to Health and Morals. By George Moore, M.D., Member of the Royal College of Physicians. *Fifth* and cheaper *Edition*. Fcp. 8vo. price 6s.

Moore.—The Use of the Body in relation to the Mind. By George Moore, M.D., Member of the Royal College of Physicians. *Third* and cheaper *Edition*. Fcp. 8vo. 6s.

Moore.—Health, Disease, and Remedy, familiarly and practically considered in a few of their relations to the Blood. By George Moore, M.D., Post 8vo. 7s. 6d.

Moore.—Memoirs, Journal, and Correspondence of Thomas Moore. Edited by the Right Hon. Lord John Russell, M.P. With Portraits and Vignette Illustrations. Vols. I. to VI. post 8vo. price 10s. 6d. each.

*** Vols. VII. and VIII., completing the work, are *nearly ready*.

Thomas Moore's Poetical Works. Containing the Author's recent Introduction and Notes. Complete in One Volume; with a Portrait, and a View of Sloperton Cottage. Medium 8vo. price 21s. cloth; morocco, 42s. Or in 10 vols. fcp. 8vo. with Portrait, and 19 Plates, price 35s.

Moore.—Songs, Ballads, and Sacred Songs. By Thomas Moore, Author of *Lalla Rookh*, &c. First collected Edition, with Vignette by R. Doyle. 16mo. price 5s. cloth; 12s. 6d. bound in morocco.

Moore's Irish Melodies. New Edition, with the Autobiographical Preface from the Collective Edition of Mr. Moore's Poetical Works, and a Vignette Title by D. Maclise, R.A. 16mo. price 5s. cloth; 12s. 6d. bound in morocco.

Moore's Irish Melodies. Illustrated by D. Maclise, R.A. New and cheaper Edition; with 161 Designs, and the whole of the Letterpress engraved on Steel, by F. P. Becker. Super-royal 8vo. price 31s. 6d. boards; bound in morocco, £2. 12s. 6d.

The Original Edition of the above, in imperial 8vo. price 63s. boards; morocco, £4. 14s. 6d.; proofs, £6. 6s. boards,—*may still be had.*

Moore's Lalla Rookh: An Oriental Romance. New Edition, with the Autobiographical Preface from the Collective Edition of Mr. Moore's Poetical Works, and a Vignette Title by D. Maclise, R.A. 16mo. price 5s. cloth; 12s. 6d. bound in morocco.

Moore's Lalla Rookh: An Oriental Romance. With 13 highly-finished Steel Plates from Designs by Corbould, Meadows, and Stephanoff, engraved under the superintendence of the late Charles Heath. New Edition. Square crown 8vo. price 15s. cloth; morocco, 28s.

*** A few copies of the Original Edition, in royal 8vo. price One Guinea, *still remain.*

Morton's Manual of Pharmacy for the Student of Veterinary Medicine: Containing the Substances employed at the Royal Veterinary College, with an attempt at their Classification; and the Pharmacopœia of that Institution. *Fifth Edition* (1854). Fcp. 8vo. price 10s.

Moseley.—The Mechanical Principles of Engineering and Architecture. By the Rev. H. Moseley, M.A., F.R.S., Professor of Natural Philosophy and Astronomy in King's College, London. 8vo. price 24s.

Mure.—A Critical History of the Language and Literature of Ancient Greece. By William Mure, M.P. of Caldwell. Vols. I. to III. 8vo. price 36s.—Vol. IV. price 15s.

"We hail with great satisfaction this continuation of a work so eminently calculated to promote the knowledge of Greek classical literature, and to increase the taste for it among the educated classes. Singularly felicitous in seizing the salient points in the character of the different tribes of the ancient Greeks, and of the different branches and periods of their literature, and in conveying to his readers,—even to those unable to follow the train of his discussion in the originals,—clear and vivid ideas on the subject, Mr. Mure has in the present volume treated in a most effective manner the general history of Greek literature during the Attic period, *i.e.*, from the usurpation of the supreme power at Athens by Pisistratus, A. 560 B.C. to the death of Alexander the Great, 323 B.C." John Bull.

Murray's Encyclopædia of Geography; Comprising a complete Description of the Earth: Exhibiting its Relation to the Heavenly Bodies, its Physical Structure, the Natural History of each Country, and the Industry, Commerce, Political Institutions, and Civil and Social State of All Nations. Second Edition; with 82 Maps, and upwards of 1,000 other Woodcuts. 8vo. price 60s.

Neale.—"Risen from the Ranks;" or, Conduct *versus* Caste. By the Rev. Erskine Neale, M.A., Rector of Kirton, Suffolk. Fcp. 8vo. price 6s.

Neale.—The Riches that bring no Sorrow. By the Rev. Erskine Neale, M.A., Rector of Kirton, Suffolk. Fcp. 8vo. price 6s.

Neale.—The Earthly Resting Places of the Just. By the Rev. Erskine Neale, M.A. Rector of Kirton, Suffolk. Fcp. 8vo. with Woodcuts, price 7s.

Neale.—The Closing Scene; or, Chris-tianity and Infidelity contrasted in the Last Hours of Remarkable Persons. By the Rev. Erskine Neale, M.A., Rector of Kirton, Suffolk. New Editions of the First and Second Series. 2 vols. fcp. 8vo. price 12s.; or separately, 6s. each.

Newman.—Discourses addressed to Mixed Congregations. By John Henry Newman, Priest of the Oratory of St. Philip Neri. Second Edition. 8vo. price 12s.

Oldacre.—The Last of the Old Squires. A Sketch. By Cedric Oldacre, Esq., of Sax-Normanbury, sometime of Christ Church, Oxon. Crown 8vo. price 9s. 6d.

Oldmixon.—Gleanings from Piccadilly to Pera. By J. W. Oldmixon, Commander R.N. With Illustrations printed in Colours. Post 8vo. [*Just ready.*

Opie (Mrs.)—Memorials of the Life of Amelia Opie, selected and arranged from her Letters, Diaries, and other Manuscripts. By Cecilia Lucy Brightwell. Second Edition; with Portrait. 8vo. price 10s. 6d.

Lieutenant Osborn's Arctic Journal. Stray Leaves from an Arctic Journal; or, Eighteen Months in the Polar Regions in Search of Sir John Franklin's Expedition. By Lieut. Sherard Osborn, R.N., Commanding H.M.S.V. *Pioneer.* With Map and Four coloured Plates. Post 8vo. price 12s.

Owen Jones.—Flowers and their Kindred Thoughts: A Series of Stanzas. By Mary Anne Bacon. With beautiful Illustrations of Flowers printed in Colours by Owen Jones. Imperial 8vo. price 31s. 6d. elegantly bound in calf.

Owen.—Lectures on the Comparative Anatomy and Physiology of the Invertebrate Animals, delivered at the Royal College of Surgeons in 1843. By Richard Owen, F.R.S., Hunterian Professor to the College. New Edition, corrected. 8vo. with Wood Engravings. [*In the press.*

Professor Owen's Lectures on the Com-parative Anatomy and Physiology of the Vertebrate Animals, delivered at the Royal College of Surgeons in 1844 and 1846. With numerous Woodcuts. Vol. I. 8vo. price 14s.

The Complete Works of Blaise Pascal. Translated from the French, with Memoir, Introductions to the various Works, Editorial Notes, and Appendices, by George Pearce, Esq. 3 vols. post 8vo. with Portrait, 25s. 6d.

VOL. 1. PASCAL'S PROVINCIAL LETTERS: with M. Villemain's Essay on Pascal prefixed, and a new Memoir. Post 8vo. Portrait, 8s. 6d.

VOL. 2. PASCAL'S THOUGHTS ON RELIGION and Evidences of Christianity, with Additions, from Original MSS.: from M. Faugère's Edition. Post 8vo. 8s. 6d

VOL. 3. PASCAL'S MISCELLANEOUS Writings, Correspondence, Detached Thoughts, &c.: from M. Faugère's Edition. Post 8vo. 8s. 6d.

Captain Peel's Travels in Nubia.—A Ride through the Nubian Desert. By Captain W. Peel, R.N. Post 8vo. with a Route Map, price 5s.

Pereira's Treatise on Food and Diet: With Observations on the Dietetical Regimen suited for Disordered States of the Digestive Organs; and an Account of the Dietaries of some of the principal Metropolitan and other Establishments for Paupers, Lunatics, Criminals, Children, the Sick, &c. 8vo. 16s.

Peschel's Elements of Physics. Trans-lated from the German, with Notes, by E. West. With Diagrams and Woodcuts. 3 vols. fcp. 8vo. 21s.

Phillips.—A Guide to Geology. By John Phillips, M.A. F.R.S. F.G.S., Deputy Reader in Geology in the University of Oxford; Honorary Member of the Imperial Academy of Sciences of Moscow, &c. Fourth Edition, corrected to the Present Time; with 4 Plates. Fcp. 8vo. price 5s.

Phillips's Elementary Introduction to Mineralogy. A New Edition, with extensive Alterations and Additions, by H. J. Brooke, F.R.S., F.G.S.; and W. H. Miller, M.A., F.G.S., Professor of Mineralogy in the University of Cambridge. With numerous Wood Engravings. Post 8vo. price 18s.

Phillips.—Figures and Descriptions of the Palæozoic Fossils of Cornwall, Devon, and West Somerset; observed in the course of the Ordnance Geological Survey of that District. By John Phillips, F.R.S. F.G.S. &c. 8vo. with 60 Plates, price 9s.

Captain Portlock's Report on the Geology of the County of Londonderry, and of Parts of Tyrone and Fermanagh, examined and described under the Authority of the Master-General and Board of Ordnance. 8vo. with 48 Plates, price 24s.

Power's Sketches in New Zealand, with Pen and Pencil. From a Journal kept in that Country, from July 1846 to June 1848. With Plates and Woodcuts. Post 8vo. 12s.

Psychological Inquiries, in a Series of Essays intended to illustrate the Influence of the Physical Organisation on the Mental Faculties. Fcp. 8vo. price 5s.

Pulman's Vade-mecum of Fly-Fishing for Trout; being a complete Practical Treatise on that Branch of the Art of Angling; with plain and copious Instructions for the Manufacture of Artificial Flies. Third Edition, with Woodcuts. Fcp. 8vo. price 6s.

Pycroft's Course of English Reading, adapted to every Taste and Capacity: With Literary Anecdotes. New and cheaper Edition. Fcp. 8vo. price 5s.

Dr. Reece's Medical Guide; for the Use of the Clergy, Heads of Families, Schools, and Junior Medical Practitioners: Comprising a complete Modern Dispensatory, and a Practical Treatise on the distinguishing Symptoms, Causes, Prevention, Cure and Palliation of the Diseases incident to the Human Frame. With the latest Discoveries in the different departments of the Healing Art, Materia Medica, &c. Seventeenth Edition, corrected and enlarged by the Author's Son, Dr. H. Reece, M.R.C.S. &c. 8vo. price 12s.

Rich's Illustrated Companion to the Latin Dictionary and Greek Lexicon: Forming a Glossary of all the Words representing Visible Objects connected with the Arts, Manufactures, and Every-day Life of the Ancients. With Woodcut Representations of nearly 2,000 Objects from the Antique. Post 8vo. price 21s.

Sir J. Richardson's Journal of a Boat Voyage through Rupert's Land and the Arctic Sea, in Search of the Discovery Ships under Command of Sir John Franklin. With an Appendix on the Physical Geography of North America; a Map, Plates, and Woodcuts. 2 vols. 8vo. price 31s. 6d.

Richardson (Captain).—Horsemanship; or, the Art of Riding and Managing a Horse, adapted to the Guidance of Ladies and Gentlemen on the Road and in the Field: With Instructions for Breaking-in Colts and Young Horses. By Captain Richardson, late of the 4th Light Dragoons. With 5 Line Engravings. Square crown 8vo. price 14s.

Riddle's Complete Latin-English and English-Latin Dictionary, for the use of Colleges and Schools. *New* and cheaper *Edition*, revised and corrected. 8vo. 21s.

Separately { The English-Latin Dictionary, 7s.
The Latin-English Dictionary, 15s.

Riddle's Copious and Critical Latin-English Lexicon, founded on the German-Latin Dictionaries of Dr. William Freund. New *and cheaper* Edition. Post 4to. 31s. 6d.

Riddle's Diamond Latin-English Dic-tionary: A Guide to the Meaning, Quality, and right Accentuation of Latin Classical Words. Royal 32mo. price 4s.

Rivers's Rose-Amateur's Guide; contain-ing ample Descriptions of all the fine leading varieties of Roses, regularly classed in their respective Families; their History and mode of Culture. Fifth Edition, corrected and improved; including a full Account of the Author's experience in the Culture of Roses in Pots. Fcp. 8vo. price 3s. 6d.

Dr. E. Robinson's Greek and English Lexicon to the Greek Testament. A New Edition, revised and in great part re-written. 8vo. price 18s.

Rogers.—Essays selected from Contributions to the Edinburgh Review. By HENRY ROGERS. 2 vols. 8vo. price 24s.

Dr. Roget's Thesaurus of English Words and Phrases Classified and arranged so as to facilitate the Expression of Ideas and assist in Literary Composition. New Edition, revised and enlarged. Medium 8vo. price 14s.

Rowton's Debater: A Series of complete Debates, Outlines of Debates, and Questions for Discussion; with ample References to the best Sources of Information on each particular Topic. New Edition. Fcp. 8vo. price 6s.

Letters of Rachel Lady Russell. A New Edition, including several unpublished Letters, together with those edited by Miss BERRY. With Portraits, Vignettes, and Facsimile. 2 vols. post 8vo. price 15s.

The Life of William Lord Russell. By the Right Hon. Lord JOHN RUSSELL, M.P. The Fourth Edition, complete in One Volume; with a Portrait engraved on Steel by S. Bellin, from the original by Sir Peter Lely at Woburn Abbey. Post 8vo. 10s. 6d.

St. John (the Hon. F.) — Rambles in Search of Sport, in Germany, France, Italy, and Russia. By the Honourable FERDINAND ST JOHN. With Four coloured Plates. Post 8vo. price 9s. 6d.

St. John (H.)—The Indian Archipelago; Its History and Present State. By HORACE ST JOHN, Author of *The British Conquests in India*, &c. 2 vols. post 8vo. price 21s.

St. John (J. A.)—There and Back Again in search of Beauty. By JAMES AUGUSTUS ST JOHN, Author of *Isis, an Egyptian Pilgrimage*, &c. 2 vols. post 8vo. price 21s.

Mr. St. John's Work on Egypt. Isis: An Egyptian Pilgrimage. By JAMES AUGUSTUS ST JOHN. 2 vols. post 8vo. 21s.

The Saints our Example. By the Author of *Letters to My Unknown Friends*, &c. Fcp. 8vo. price 7s.

Schmitz.—History of Greece, from the Earliest Times to the Taking of Corinth by the Romans, B.C. 146, mainly based upon Bishop Thirlwall's History of Greece. By Dr. LEONHARD SCHMITZ, F.R.S.E., Rector of the High School of Edinburgh. New Edition. 12mo. price 7s. 6d.

Sir Edward Seaward's Narrative of his Shipwreck, and consequent Discovery of certain Islands in the Caribbean Sea. Third Edition. 2 vols. post 8vo. 21s.—An ABRIDGMENT, in 16mo. price 2s. 6d.

The Sermon in the Mount. Printed by C. Whittingham, uniformly with the *Thumb Bible*; bound and clasped. 64mo. price Eighteenpence.

The Sermon on the Mount. Printed on Silver; with Picture Subjects, numerous Landscape and Illustrative Vignettes, and Illuminated Borders in Gold and Colours, designed by M. LEPELLE DU BOIS-GALLAIS. Square 18mo. price in ornamental boards, One Guinea; or 31s. 6d. bound in morocco.

Self-Denial the Preparation for Easter. By the Author of *Letters to my Unknown Friends*, &c. Fcp. 8vo. price 2s. 6d.

Sharp's New British Gazetteer, or Topo-graphical Dictionary of the British Islands and Narrow Seas: Comprising concise Descriptions of about Sixty Thousand Places, Seats, Natural Features, and Objects of Note, founded on the best Authorities; full Particulars of the Boundaries, Registered Electors, &c. of the Parliamentary Boroughs; with a reference under every name to the Sheet of the Ordnance Survey, as far as completed; and an Appendix, containing a General View of the Resources of the United Kingdom, a Short Chronology, and an Abstract of certain Results of the Census of 1851. 2 vols. 8vo. price £2. 16s.

Sewell. — Amy Herbert. By a Lady. Edited by the Rev. WILLIAM SEWELL, B.D. Fellow and Tutor of Exeter College, Oxford. New Edition. Fcp. 8vo. price 6s.

Sewell.—The Earl's Daughter. By the Author of *Amy Herbert*. Edited by the Rev. W. SEWELL, B.D. 2 vols. fcp. 8vo. 9s.

Sewell. — Gertrude: A Tale. By the Author of *Amy Herbert*. Edited by the Rev. W. SEWELL, B.D. New Edition. Fcp. 8vo. price 6s.

Sewell.—Laneton Parsonage: A Tale for Children, on the Practical Use of a portion of the Church Catechism. By the Author of *Amy Herbert*. Edited by the Rev. W. SEWELL, B.D. New Edition. 3 vols. fcp. 8vo. price 16s.

Sewell. — Margaret Percival. By the Author of *Amy Herbert*. Edited by the Rev. W. SEWELL, B.D. New Edition. 2 vols. fcp. 8vo. price 12s.

By the same Author,

Katharine Ashton. New Edition. 2 vols. fcp. 8vo. price 12s.

The Experience of Life. New Edition. Fcp. 8vo. price 7s. 6d.

Readings for a Month preparatory to Confirmation: Compiled from the Works of Writers of the Early and of the English Church. Fcp. 8vo. price 5s. 6d.

Readings for Every Day in Lent: Compiled from the Writings of BISHOP JEREMY TAYLOR. Fcp. 8vo. price 5s.

The Family Shakspeare; in which nothing is *added* to the Original Text; but those words and expressions are *omitted* which cannot with propriety be read aloud. By T. BOWDLER, Esq. F.R.S. New Edition, in Volumes for the Pocket; with 36 Wood Engravings, from Designs by Smirke, Howard, and other Artists. 6 vols. fcp. 8vo. price 30s.

*** Also a LIBRARY EDITION, in One Volume, medium 8vo. price 21s.

Short Whist; Its Rise, Progress, and Laws: With Observations to make any one a Whist Player. Containing also the Laws of Piquet, Cassino, Ecarté, Cribbage, Backgammon. By Major A *****. New Edition; to which are added, Precepts for Tyros, by Mrs. B ****. Fcp. 8vo. 3s.

Sinclair. — The Journey of Life. By CATHERINE SINCLAIR, Author of *The Business of Life* (2 vols. fcp. 8vo. price 10s.) New Edition, corrected and enlarged. Fcp. 8vo. price 5s.

Sir Roger de Coverley. From The Spectator. With Notes and Illustrations, by W. HENRY WILLS; and Twelve fine Wood Engravings, by John Thompson, from Designs by FREDERICK TAYLER. Crown 8vo. price 15s. boards; or 27s. bound in morocco.—A Cheap Edition, without Woodcuts, in 16mo. price One Shilling.

Smee's Elements of Electro-Metallurgy. Third Edition, revised, corrected, and considerably enlarged; with Electrotypes and numerous Woodcuts. Post 8vo. price 10s. 6d.

Smith's Sacred Annals.—Sacred Annals: Vol. III. The Gentile Nations; or, The History and Religion of the Egyptians, Assyrians, Babylonians, Medes, Persians, Greeks, and Romans, collected from ancient authors and Holy Scripture, and including the recent discoveries in Egyptian, Persian, and Assyrian Inscriptions: Forming a complete connection of Sacred and Profane History, and shewing the Fulfilment of Sacred Prophecy. By GEORGE SMITH, F.A.S. &c. In Two Parts, crown 8vo. price 12s.

By the same Author,

Sacred Annals: Vol. I. The Patriarchal Age; or, Researches into the History and Religion of Mankind, from the Creation of the World to the Death of Isaac. Crown 8vo. 10s.

Sacred Annals: Vol. II. The Hebrew People; or, The History and Religion of the Israelites, from the Origin of the Nation to the Time of Christ. In two Parts, crown 8vo. price 12s.

The Rev. Sydney Smith's Miscellaneous Works: Including his Contributions to The Edinburgh Review. Three Editions, as follows:—

1. LIBRARY EDITION (the *Fourth*), in 3 vols. 8vo. with Portrait, price 36s.
2. Complete in ONE VOLUME, with Portrait and Vignette. Square crown 8vo. price 21s. cloth; or 30s. calf.
3. A NEW EDITION, in 3 vols. fcp. 8vo. price 21s.

The Rev. Sydney Smith's Elementary Sketches of Moral Philosophy, delivered at the Royal Institution in the Years 1804, 1805, and 1806. Second Edition. 8vo. price 12s.

The Life and Correspondence of the late Robert Southey. Edited by his Son, the Rev. C. C. SOUTHEY, M.A., Vicar of Ardleigh. With Portraits, and Landscape Illustrations. 6 vols. post 8vo. price 63s.

Southey's Life of Wesley; and Rise and Progress of Methodism. New Edition, with Notes and Additions. Edited by the Rev. C. C. SOUTHEY, M.A. 2 vols. 8vo. with 2 Portraits, price 28s.

Southey's Commonplace Books. Comprising — 1. Choice Passages: With Collections for the History of Manners and Literature in England; 2. Special Collections on various Historical and Theological Subjects; 3. Analytical Readings in various branches of Literature; and 4. Original Memoranda, Literary and Miscellaneous. Edited by the Rev. J. W. WARTER, B.D. 4 vols. square crown 8vo. price £3. 18s.

Each *Commonplace Book*, complete in itself, may be had separately as follows:—

FIRST SERIES—CHOICE PASSAGES, &c. 18s.
SECOND SERIES—SPECIAL COLLECTIONS. 18s.
THIRD SERIES—ANALYTICAL READINGS. 21s.
FOURTH SERIES—ORIGINAL MEMORANDA, &c. 21s.

Southey's The Doctor &c. Complete in One Volume. Edited by the Rev. J. W. WARTER, B.D. With Portrait, Vignette, Bust, and coloured Plate. New Edition. Square crown 8vo. price 21s.

Robert Southey's Complete Poetical Works; containing all the Author's last Introductions and Notes. Complete in One Volume, with Portrait and Vignette. Medium 8vo. price 21s. cloth; 42s. bound in morocco. Or in 10 vols. fcp. 8vo. with Portrait and 19 Plates, price 35s.

Select Works of the British Poets; from Chaucer to Lovelace, inclusive. With Biographical Sketches by the late ROBERT SOUTHEY. Medium 8vo. price 30s.

Stephen.—Lectures on the History of France. By the Right Hon. Sir JAMES STEPHEN, K.C.B. LL.D. Professor of Modern History in the University of Cambridge. Second Edition. 2 vols. 8vo. price 24s.

Stephen.—Essays in Ecclesiastical Biography; from The Edinburgh Review. By the Right Hon. Sir JAMES STEPHEN, K.C.B. LL.D. Third Edition. 2 vols. 8vo. 24s.

Stonehenge.—The Greyhound: Being a Treatise on the Art of Breeding, Rearing, and Training Greyhounds for Public Running; their Diseases and Treatment: Containing also, Rules for the Management of Coursing Meetings, and for the Decision of Courses. By STONEHENGE. With numerous Portraits of Greyhounds, &c. engraved on Wood, and a Frontispiece engraved on Steel. Square crown 8vo. price 21s.

Stow.—The Training System, the Moral Training School, and the Normal Seminary for preparing School-Trainers and Governesses. By DAVID STOW, Esq., Honorary Secretary to the Glasgow Normal Free Seminary. Tenth Edition; with Plates and Woodcuts. Post 8vo. price 6s.

Dr. Sutherland's Journal of a Voyage in Baffin's Bay and Barrow's Straits, in the Years 1850 and 1851, performed by H.M. Ships *Lady Franklin* and *Sophia*, under the command of Mr. William Penny, in search of the missing Crews of H.M. Ships *Erebus* and *Terror*. With Charts and Illustrations. 2 vols. post 8vo. price 27s.

Tate.—On the Strength of Materials; Containing various original and useful Formulæ, specially applied to Tubular Bridges, Wrought Iron and Cast Iron Beams, &c. By THOMAS TATE, F.R.A.S. 8vo. price 5s. 6d.

Taylor.—Loyola: And Jesuitism in its Rudiments. By ISAAC TAYLOR. Post 8vo. with Medallion, price 10s. 6d.

Taylor.—Wesley and Methodism. By ISAAC TAYLOR. Post 8vo. with a Portrait, price 10s. 6d.

Theologia Germanica: Which setteth forth many fair lineaments of Divine Truth, and saith very lofty and lovely things touching a Perfect Life. Translated by SUSANNA WINKWORTH. With a Preface by the Rev. CHARLES KINGSLEY; and a Letter by Chevalier BUNSEN. Fcp. 8vo. price 5s.

Thirlwall.—The History of Greece. By the Right Rev. the LORD BISHOP of ST. DAVID'S (the Rev. Connop Thirlwall). An improved Library Edition; with Maps. 8 vols. 8vo. price £4. 16s.

*** Also, an Edition in 8 vols. fcp. 8vo. with Vignette Titles, price 28s.

Thomson (The Rev. W.)—An Outline of the Laws of Thought: Being a Treatise on Pure and Applied Logic. By the Rev. W. THOMSON, M.A. Fellow and Tutor of Queen's College, Oxford. Third Edition, enlarged. Fcp. 8vo. price 7s. 6d.

Thomson's Tables of Interest, at Three, Four, Four-and-a-Half, and Five per Cent., from One Pound to Ten Thousand, and from 1 to 365 Days, in a regular progression of single Days; with Interest at all the above Rates, from One to Twelve Months, and from One to Ten Years. Also, numerous other Tables of Exchanges, Time, and Discounts. New Edition. 12mo. price 8s.

Thomson's Seasons. Edited by Bolton Corney, Esq. Illustrated with Seventy-seven fine Wood Engravings from Designs by Members of the Etching Club. Square crown 8vo. price 21s. cloth; or, 36s. bound in morocco.

Thornton.—Zohrab; or, a Midsummer Day's Dream: And other Poems. By William Thomas Thornton. Fcp. 8vo. price 2s. 6d.

The Thumb Bible; or, Verbum Sempiternum. By J. Taylor. Being an Epitome of the Old and New Testaments in English Verse. Reprinted from the Edition of 1693; bound and clasped. 64mo. 1s. 6d.

Todd (Charles).—A Series of Tables of the Area and Circumference of Circles; the Solidity and Superficies of Spheres; the Area and Length of the Diagonal of Squares; and the Specific Gravity of Bodies, &c.: To which is added, an Explanation of the Author's Method of Calculating these Tables. Intended as a Facility to Engineers, Surveyors, Architects, Mechanics, and Artisans in general. By Charles Todd, Engineer. The Second Edition, improved and extended. Post 8vo. price 6s.

Townsend.—The Lives of Twelve Eminent Judges of the Last and of the Present Century. By W. C. Townsend, Esq., M.A., Q.C. 2 vols. 8vo. price 28s.

Townsend.—Modern State Trials revised and illustrated with Essays and Notes. By W. C. Townsend, Esq. M.A. Q.C. 2 vols. 8vo. price 30s.

Sharon Turner's Sacred History of the World, attempted to be Philosophically considered, in a Series of Letters to a Son. New Edition, edited by the Author's Son, the Rev. S. Turner. 3 vols. post 8vo. price 31s. 6d.

Sharon Turner's History of England during the Middle Ages: Comprising the Reigns from the Norman Conquest to the Accession of Henry VIII. Fifth Edition, revised by the Rev. S. Turner. 4 vols. 8vo. price 50s.

Sharon Turner's History of the Anglo-Saxons, from the Earliest Period to the Norman Conquest. The Seventh Edition, revised by the Rev. S. Turner. 3 vols. 8vo. price 36s.

Dr. Turton's Manual of the Land and Fresh-water Shells of the British Islands. A New Edition, with considerable Additions by John Edward Gray: With Woodcuts, and 12 coloured Plates. Post 8vo price 15s.

Dr. Ure's Dictionary of Arts, Manufac-tures, and Mines: Containing a clear Exposition of their Principles and Practice. The Fourth Edition, much enlarged and corrected throughout; with all the Information comprised in the *Supplement of Recent Improvements* brought down to the Present Time and incorporated in the *Dictionary*. Most of the Articles being entirely re-written, and many new Articles now first added. With nearly 1,600 Woodcuts. 2 vols 8vo. price 60s.

Waterton.—Essays on Natural History, chiefly Ornithology. By C. Waterton, Esq. With an Autobiography of the Author and Views of Walton Hall. New and cheaper Edition. 2 vols fcp 8vo price 10s.

Separately. Vol I. (First Series), 5s. 6d. Vol II. (Second Series), 4s. 6d.

Alaric Watts's Lyrics of the Heart, and other Poems. With 41 highly-finished Line Engravings, executed expressly for the work by the most eminent Painters and Engravers. Square crown 8vo. price 31s 6d. boards, or 45s. bound in morocco; Proof Impressions, 63s. boards.

Webster and Parkes's Encyclopædia of Domestic Economy; Comprising such subjects as are most immediately connected with Housekeeping: As, The Construction of Domestic Edifices, with the modes of Warming, Ventilating, and Lighting them—A description of the various articles of Furniture, with the nature of their Materials—Duties of Servants, &c. New Edition; with nearly 1,000 Woodcuts. 8vo. price 50s.

PUBLISHED BY LONGMAN, BROWN, AND CO.

THE TRAVELLER'S LIBRARY,

IN COURSE OF PUBLICATION IN VOLUMES PRICE HALF-A-CROWN EACH;

Comprising books of valuable information and acknowledged merit, in a form adapted for reading while Travelling, and also of a character that will render them worthy of preservation.

List of the VOLUMES *already published.*

VOL. I	MACAULAY's ESSAYS on WARREN HASTINGS and LORD CLIVE	2/6
II.	——— ESSAYS on PITT and CHATHAM, RANKE and GLADSTONE,	2/6
III.	LAING's RESIDENCE in NORWAY	2/6
IV.	PFEIFFER's VOYAGE ROUND the WORLD	2/6
V.	EOTHEN, TRACES of TRAVEL from the EAST	2/6
VI	MACAULAY's ESSAYS on ADDISON, WALPOLE, and LORD BACON	2/6
VII.	HUC's TRAVELS in TARTARY, &c.	2/6
VIII	THOMAS HOLCROFT's MEMOIRS	2/6
IX.	WERNE's AFRICAN WANDERINGS	2/6
X	MRS JAMESON's SKETCHES in CANADA	2/6
XI	JERRMANN's PICTURES from ST. PETERSBURG	2/6
XII	THE REV G R. GLEIG's LEIPSIC CAMPAIGN	2/6
XIII	HUGHES's AUSTRALIAN COLONIES	2/6
XIV	SIR EDWARD SEAWARD's NARRATIVE	2/6
XV.	ALEXANDRE DUMAS' MEMOIRS of a MAITRE D'ARMES	2/6
XVI	OUR COAL FIELDS and OUR COAL PITS	2/6
XVII	M'CULLOCH's LONDON and GIRONIÈRE's PHILIPPINES	2/6
XVIII	SIR ROGER DE COVERLEY and SOUTHEY's LOVE STORY	2/6
XIX	LORD CARLISLE's LECTURES and ADDRESSES, and JEFFREY's ESSAYS on SWIFT and RICHARDSON	2/6
XX	HOPE'S BIBLE in BRITTANY and CHASE in BRITTANY	2/6
XXI	THE ELECTRIC TELEGRAPH and NATURAL HISTORY of CREATION,	2/6
XXII.	MEMOIR of DUKE of WELLINGTON and LIFE of MARSHAL TURENNE,	2/6
XXIII	RANKE's FERDINAND and MAXIMILIAN & TURKEY and CHRISTENDOM	2/6
XXIV	BARROW's CONTINENTAL TOUR and FERGUSON's SWISS MEN and SWISS MOUNTAINS	2/6
XXV.	SOUVESTRE's ATTIC PHILOSOPHER in PARIS, and WORKING MAN'S CONFESSIONS	
XXVI.	MACAULAY's ESSAYS on LORD BYRON and the COMI[illegible] and his SPEECHES on PARLIAMENTARY R[illegible] (1831-32)	2/6
XXVII	SHIRLEY BROOKS'S RUSSIANS of the SOUTH; and DR. KEMP'S INDICATIONS of INSTINCT	2/6
XXVIII.	LANMAN'S ADVENTURES in the WILDS of NORTH AMERICA	2/6
XXIX	DE CUSTINE's RUSSIA, abridged	2/6
XXX.	SELECTIONS from SYDNEY SMITH's WRITINGS, Vol. I.	2/6
XXXI.	BODENSTEDT and WAGNER's SCHAMYL; and M'CULLOCH's RUSSIA and TURKEY	2/6
XXXII.	LAING's NOTES of a TRAVELLER, First Series	2/6
XXXIII.	DURRIEU'S MOROCCO; and an ESSAY on MORMONISM	2/6

THE TRAVELLER'S LIBRARY,

In course of publication in Parts price One Shilling each: Comprising books of valuable information and acknowledged merit, in a form adapted for reading while Travelling, and also of a character that will render them worthy of preservation

List of the PARTS *already published*:—

1 Mr Macaulay's Essay on Warren Hastings
2 " " Lord Clive
3 London in the years 1850 and 1851 By J R M'Culloch
4 Sir Roger De Coverley From the Spectator
5 Mr Macaulay's Two Essays on William Pitt and the Earl of Chatham
6 7 Laing's Residence in Norway
8 Mr Macaulay's Essays on Ranke and Gladstone
9—10 Ida Pfeiffer's Lady's Voyage round the World
11—12 Eothen, or Traces of Travel from the East
13 Mr Macaulay's Essays on Addison and Walpole
14—15 Huc's Travels in Tartary Thibet, and China
16—17 Thomas Holcroft's Memoirs
18 The Earl of Carlisle's Lectures and Addresses
19—20 Werne's African Wanderings
21—22 Mrs Jameson's Sketches in Canada
23 Brittany and the Bible By I Hope
24 The Natural History of Creation By Dr L Kemp
25 Mr Macaulay's Essay on Lord Bacon
26 The Electric Telegraph, etc By Dr G Wilson
27—28 Jerrmann's Pictures from St. Petersburg
29—30 The Rev G R Gleig's Leipsic Campaign
31 Memoir of the Duke of We[llingto]n
32—33 The Australian Colonies [By Wil]liam Hughes F R G S
34—35 Sir Edward Seaward's Narr[ative, ab]ridged
36 Lord Jeffrey's Essays on Swi[ft and Ric]hardson
37 Ranke's Ferdinand I and M[aximilia]n II
38—39 Memoirs of a Maitre d'Armes [By A]lexandre Dumas
40 Byron and the Comic Dramatists [B]y T B Macaulay
41 Marshal Turenne By the Rev [T O] Cockayne, M A
42—43 Our Coal Fields and Coal Pits
44 Barrow's Tour on the Continent
45 Swiss Men and Swiss Mountains
46 Gironière's Philippine Islands
47 Turkey and Christendom
48 Confessions of a Working Man B[y E] Souvestre
49 The Chase in Brittany By I Hope
... res of ... y from Southey's Docto[r]
53 The Ru[ssian] ... By F ...
54 Indications of Instinct By Dr [Lindley Kemp]
55—56 Lanman's Adventures in the W[ilds of North America]
57 58 59 De Custine's Russia
60 Durrieu's Morocco
61—62 Selections from Sydney Smith's [Writings]
63 Schamyl the Chieftain of the Cau[casus]
64 Russia and Turkey By J R M'C[ulloch]
65—66 Laing's Notes of a Traveller First [Series]
67 Mormonism

Wheeler.—The Geography of Herodotus Developed, Explained, and Illustrated from Modern Researches and Discoveries By J TALBOYS WHEELER, F.R.G S. 8vo with Maps and Plans [*Nearly ready*

Willich's Popular Tables for ascertaining the Value of Lifehold, Leasehold, and Church Property, Renewal Fines, &c Third Edition, with additional Tables of Natural or Hyperbolic Logarithms, Trigonometry, Astronomy, Geography, &c Post 8vo price 9s

Lady Willoughby's Diary (1635 to 1663). Printed, ornamented, and bound in the style of the period to which *The Diary* refers New Edition, in Two Parts Square fcp 8vo price 8s each, boards, or, bound in morocco, 18s each

Wilmot's Abridgment of Blackstone's Commentaries on the Laws of England, intended for the use of Young Persons, and comprised in a series of Letters from a Father to his Daughter. A New Edition, corrected and brought down to the Present Day, by SIR JOHN E. EARDLEY WILMOT, Bart 12mo price 6s 6d

Yonge.—A New English-Greek Lexicon Containing all the Greek Words used by Writers of good authority By C D YONGE, B A Post 4to price One Guinea

Yonge's New Latin Gradus: Containing every Word used by the Poets of good authority By Authority and for the Use of Eton, Westminster, Winchester, Harrow, Charterhouse, and Rugby Schools King's College, London, and Marlborough College, Third Edition, carefully revised and corrected Post 8vo price 9s

Youatt.—The Horse. By William Youatt. With a Treatise of Draught New Edition, with numerous Wood Engravings, from Designs by William Harvey (Messrs Longman and Co's Edition should be ordered.) 8vo price 10s

Youatt.—The Dog. By William Youatt A New Edition; with numerous Engravings, from Designs by W. Harvey. 8vo 6s

Zumpt's Grammar of the Latin Language Translated and adapted for the use of English Students by Dr L. SCHMITZ, F.R.S.E.: With numerous Additions and Corrections by the Author and Translator The Third Edition, thoroughly revised, with an Index 8vo price 14s

[*September*, 1854

WILSON AND OGILVY, SKINNER STREET, SNOW HILL, LONDON.